DOCUMENT RESUME

ED 276 628 SE 047 632

TITLE	Mathematics Framework, Kindergarten-Grade 12.
INSTITUTION	Texas Education Agency, Austin.
PUB DATE	Nov 86
NOTE	143p.
AVAILABLE FROM	Publications Distributions Office, Texas Education Agency, 1701 North Congress Avenue, Austin, TX 78701-1494 (Initial copy free, additional copies, $3.00).
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC06 Plus Postage.
DESCRIPTORS	*Course Descriptions; *Educational Objectives; Educational Philosophy; Elementary Secondary Education; *Guidelines; *Mathematics Curriculum; *Mathematics Education; *Mathematics Instruction; Problem Solving
IDENTIFIERS	*Texas

ABSTRACT

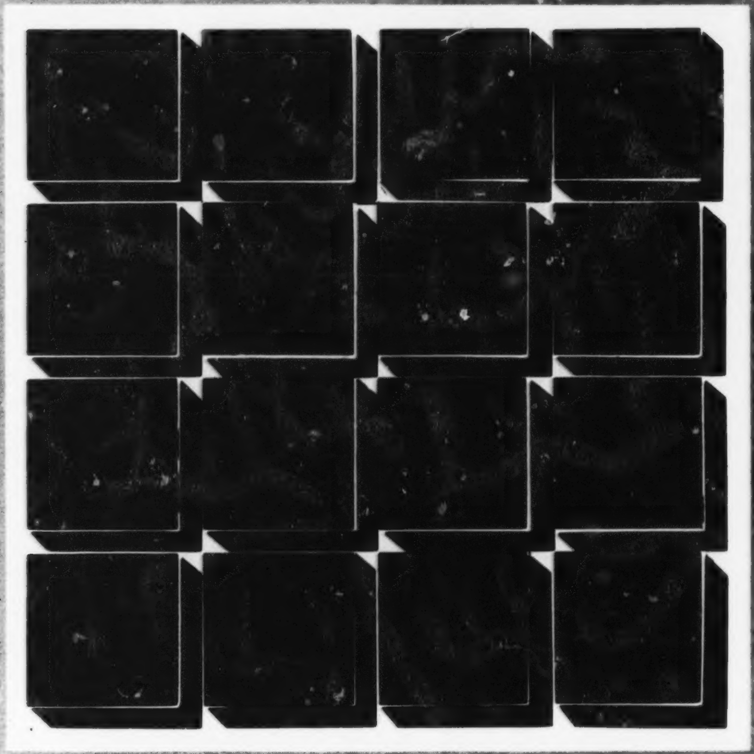
This publication should help educators provide a mathematics program that emphasizes productive time on task and active involvement of students in mathematics activities. The focus on problem solving is stressed. Time allotments are stated, followed by descriptions of essential elements for kindergarten through grade 8: understanding numbers and place value; operations, properties, and uses of numbers; problem solving; measurement; geometry; graphing; probability and statistics; and calculators and computers. Essential elements of each secondary school mathematics course are then presented. The next section provides correlations of the essential elements to the Texas Educational Assessment of Minimum Skills tests. A scope-and-sequence chart is included, as are discussions of evaluation, reteaching and remediation, enrichment, assignment of teachers, and staff development. In the next section, the philosophy of the elementary and middle school mathematics programs is discussed, with recommended manipulative materials listed. The secondary school mathematics program is then considered, with philosophy, graduation requirements, approved courses, planning course offerings, guidelines for course sequencing, and honors courses discussed. Special student populations are next considered. Appendices contain position statements, references, and some suggested materials. (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED276628

MATHEMATICS FRAMEWORK

KINDERGARTEN – GRADE 12



TEXAS EDUCATION AGENCY   AUSTIN, TEXAS

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

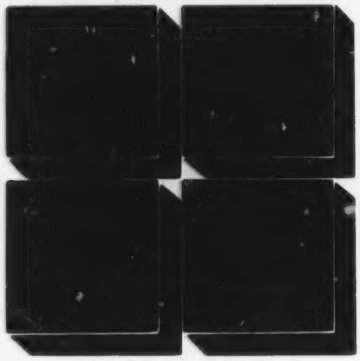
- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Linda
Kemp
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 047 653

BEST COPY AVAILABLE

Mathematics Framework, Kindergarten-Grade 12



This publication is not copyrighted; any or all sections may be duplicated.

After an initial free distribution to authorized institutions, additional copies may be purchased for $3.00 from the Publications Distribution Office.

Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701-1494

STATE BOARD OF EDUCATION

(State Board for Vocational Education)

JON BRUMLEY, Fort Worth
Chairman of the State Board of Education
District 11

REBECCA CANNING, Waco
Vice Chairman of the State Board of Education
District 9

EMMETT J. CONRAD, Dallas
Secretary of the State Board of Education
District 13

Board Members

VOLLY C. BASTINE, JR., Houston
District 4

MARY HELEN BERLANGA, Corpus Christi
District 2

CAROLYN HONEA CRAWFORD, Beaumont
District 7

CHARLES W. DUNCAN, JR., Houston
District 6

PAUL C. DUNN, Levelland
District 15

MARIA ELENA A. FLOOD, El Paso
District 1

WILLIAM V. McBRIDE, San Antonio
District 5

GERALDINE MILLER, Dallas
District 12

PETE MORALES, JR., Devine
District 3

JOHN MACK PRESCOTT, College Station
District 10

KATHERINE PEARCY RAINES, Cleburne
District 14

JACK STRONG, Longview
District 8

W. N. KIRBY, Commissioner of Education
(Executive Officer of the State Board of Education)

Committees
of the
State Board of Education

Personnel
JOHN MACK PRESCOTT, Chairman
REBECCA CANNING
EMMETT J. CONRAD
CAROLYN HONEA CRAWFORD
PAUL C. DUNN

Long-Range Planning
REBECCA CANNING, Chairman
MARY HELEN BERLANGA
EMMETT J. CONRAD
CAROLYN HONEA CRAWFORD
CHARLES W. DUNCAN, JR.
WILLIAM V. McBRIDE

Finance & Programs
MARIA ELENA A. FLOOD, Chairman
JON BRUMLEY
WILLIAM V. McBRIDE
GERALDINE MILLER
PETE MORALES, JR.

Students
JACK STRONG, Chairman
VOLLY C. BASTINE, JR.
MARY HELEN BERLANGA
CHARLES W. DUNCAN, JR.
KATHERINE PEARCY RAINES

TEXAS EDUCATION AGENCY

This publication was developed by Agency staff members in the Office of the Deputy Commissioner for Curriculum and Program Development.

VICTORIA BERGIN
Deputy Commissioner for Curriculum
and Program Development

DELIA POMPA
Assistant Commissioner for Program
Development

GRACE GRIMES
Assistant Commissioner for Curriculum
and Instruction

LEROY F. PSENCIK
Director, Division of Curriculum Development

Mathematics Section

CATHY SEELEY PEAVLER
Director of Mathematics

RAMONA JO DeVALCOURT
Mathematics Specialist, Elementary

BILLY L. HOPKINS
Mathematics Specialist, High School

BARBARA MONTALTO
Mathematics Specialist, Middle School
and Junior High

VICKY SCHREIBER DILL
Curriculum Writer
Division of Curriculum Development

PAT FOLMAR ROBINSON
Educational Writer
Division of Curriculum Development

Foreword

This curriculum publication, *Mathematics Framework, Kindergarten-Grade 12,* is designed to help administrators and teachers make mathematics the important part of the well-balanced Texas curriculum it must be. Mathematics is to be given significant instructional attention each school day, according to mandates of the Texas Legislature and to State Board of Education rules.

The framework should help educators provide a mathematics program that emphasizes, at all levels of instruction, productive time on task. It should help educators actively involve students in structured activities that will develop their understanding and ability to apply skills in a variety of problem situations.

W. N. Kirby

Commissioner of Education

Contents

Overview of the Mathematics Program

Introduction 3

The Focus of the Mathematics Program: Problem Solving 4

Time Allotments in Mathematics, K-Grade 8 5

Content Description of Essential Elements, K-Grade 8 6

Essential Elements

Kindergarten 12
Grade 1 13
Grade 2 14
Grade 3 15
Grade 4 16
Grade 5 18
Grade 6 20
Grade 7 22
Grade 8 24
Fundamentals of Mathematics 26
Consumer Mathematics 28
Pre-Algebra 30
Informal Geometry 32
Algebra I 35
Geometry 37
Algebra II 39
Trigonometry 41
Elementary Analysis 42
Analytic Geometry 43
Precalculus 44
Calculus 46
Advanced Mathematics for Business 47
Mathematics of Consumer Economics 49
Computer Mathematics I 50
Computer Mathematics II 52

Probability and Statistics 54
History of Mathematics 55
Survey of Mathematics 57
Number Theory 58
Linear Algebra 59
Linear Programming 60

Correlation of Essential Elements to TEAMS Objectives 61

Evaluation 70

Scope and Sequence Chart 71

Reteaching and Remediation 75

Enrichment 77

Assignment of Teachers 78

Staff Development in Mathematics 79

The Challenge 81

Elementary and Middle School Mathematics

Philosophy of the Elementary Mathematics Program 85

Philosophy of the Mathematics Program, Grades 7-8 86

Recommended Minimum Mathematics Manipulative Materials 87

High School Mathematics

Philosophy of Mathematics Program, Grades 9-12 93

Mathematics Requirements for High School Graduation 94

Mathematics Courses, Grades 9-12 95

Planning Course Offerings to Meet Students, Needs 96
Options for Small Schools 96
Summary of High School Mathematics Courses 98

Recommended Guidelines for Course Sequencing 104

Recommended Three-Year Sequences 105

Honors Courses 106

Special Student Populations and Mathematics

Introduction 109

Goals 110

Special Education 111

Gifted and Talented Education 115

Special Programs 116
  Compensatory Education 116
  Bilingual Education 117
  Migrant Education 120

Appendices

Professional Development Programs for Teachers of Mathematics (NCTM) 125

Calculators in the Mathematics Classroom (NCTM) 127

Selected References: Encouraging Full Participation in Mathematics 128

Suggested Materials for Teaching Informal Geometry 130

Suggested Materials for Teaching Advanced Mathematics for Business 132

Overview of the Mathematics Program

Introduction

The mathematics program in Texas reflects what many educators recognize as appropriate mathematics instruction for students. It presents instructional strategies that help students develop their mathematical talents to the fullest. The program deliberately focuses on solving problems and teaching for understanding.

This framework includes State Board of Education rules related to mathematics and guidelines for implementing an effective mathematics program. The State Board rules are contained in Title 19, Chapter 75 of the Texas Administrative Code. In addition to Chapter 75 rules, this framework includes other information about mathematics such as specific time allocations for elementary grades, a definition of mastery of the essential elements, a list of courses required to be taught in high school, and high school graduation requirements. The framework presents suggestions for sequencing secondary courses, prerequisite information, placement criteria, and a philosophy of teaching strategies and methodologies. In addition, general information is given concerning assessment, suggestions for reteaching and remediation, and recommended manipulative materials for K-Grade 8. A major section provides curriculum modifications for teaching mathematics to special student populations.

The Focus of the Mathematics Program: Problem Solving

The major focus of the mathematics program is to teach students to use mathematics to solve problems. Consensus that this should be the program's principle aim was reached by the broad range of Texas educators who assisted in developing the Chapter 75 mathematics program. The thinking of these Texas educators is supported by the National Council of Teachers of Mathematics which identified problem solving as the major thrust of mathematics education for the 1980s.

While problem solving is identified as a separate essential element, the teaching of problem solving cannot be viewed as an isolated skill. Rather, problem solving is an underlying approach to the teaching of all mathematics. Computation is not taught as an end in itself but as an extension of real-life situations. Students must experience the opportunity to think critically, to analyze, and to apply skills and concepts to both routine and nonroutine mathematical problems.

Our rapidly changing technological society requires students to be able to go far beyond the limits of what has been traditionally taught in mathematics classrooms. To have the opportunity to exercise higher order thinking skills in emerging fields in the future, Texas students must have education today that prepares them to use all of the problem-solving tools and strategies available.

Estimation

Students must be taught estimation skills to predict and validate answers to problems. The knowledge that answers are reasonable or unreasonable is extremely important as students analyze problems and their solutions. In general, estimates of measurements can simplify the teaching of otherwise rote skills.

Calculators

The calculator is a powerful problem-solving tool the accurate use of which requires an understanding of computation concepts. Consequently, its use does not replace the teaching of computation. While the calculator may even have a role in the teaching of computation itself, its main function can be to free students from lengthy and tedious computations when they are incidental to the problem being investigated. Students can use a calculator in solving sophisticated real problems with large or small numbers. Such problems might never be approachable without this tool.

The use of calculators is specified by essential elements in Grades 7 and 8. Their use in problem solving is also recommended in elementary and middle school and in all high school mathematics courses. Teachers should provide students with experiences using calculators through both the instruction and evaluation of problem-solving skills.

Computers

The computer is evolving as a rapidly changing, highly flexible, problem-solving tool. Whereas at one time the computer was used primarily to teach programming, its function in mathematics education has broadened.

The computer can be used to make predictions, to validate possible solutions to problems, to investigate properties and concepts of mathematics, to perform lengthy repetitive procedures, and to simulate situations. In years to come, there undoubtedly will be many other applications currently not even imagined. Students, therefore, should have access to the computer as a problem-solving tool in all mathematics courses at the elementary, middle school, and high school levels.

Time Allotments in Mathematics, K-Grade 8

KINDERGARTEN: 20 percent of the instructional day must be spent on mathematics.

ELEMENTARY (GRADES 1-5 or 1-6): Mathematics must be taught daily for a minimum of 300 minutes per week.

MIDDLE SCHOOL (GRADES 6-8) OR JUNIOR HIGH SCHOOL (GRADES 7-8): 45 minutes per day, exclusive of passing periods, constitutes a unit of mathematics.

HIGH SCHOOL: The equivalent of 55 minutes per day, exclusive of passing periods, constitutes a unit of mathematics.

All levels of mathematics instruction must include active involvement of students and stress experiences designed to teach understanding and application of concepts and skills.

Content Description of Essential Elements, K-Grade 8

The essential elements and subelements of the statewide mathematics curriculum are designed to allow some interpretation by school district personnel to reflect student need within the district. It is the responsibility of school district personnel to formulate objectives that describe how the teaching of the essential elements will take place. For this reason, essential elements are not stated as specific objectives. The purpose of this section is to give general content guidelines to accompany the essential elements.

The essential elements are delineated with the assumption that students have mastered appropriate prerequisite skills. For the first few years of implementation of this curriculum, teachers may have to devote considerable teaching time to reviewing prerequisite skills. This may make it difficult to cover all the essential elements described for the year. In the early years of implementation, some essential elements may not be taught in as great a depth as districts desire. However, as students become better prepared and master appropriate prerequisite skills, there should be adequate time in the school year to cover each essential element thoroughly.

Teachers should prioritize topics so that they do not overteach introductory and review objectives. They should focus their time on those essential elements that are the main responsibility of the level being taught. Introductory objectives and review should encompass only a small part of the daily instructional time.

Mathematics Essential Elements, K-Grade 8

- Understanding numbers and place value
- Operations on numbers, properties of numbers, and uses of numbers
- Problem solving
- Measurement
- Geometry
- Graphing
- Probability and statistics
- Calculators and computers (Grades 7-8)

Essential Element 1: Number Concepts

In the mathematics program, Essential Element 1 addresses the concepts and skills associated with the understanding of numbers. These numbers include whole, integer, and nonnegative rational numbers. The element also addresses an understanding of the place-value system.

Kindergarten students experience concepts involving number at the preparation or readiness level. The instructor introduces them to number concepts through use of manipulatives which are an integral component of readiness activities. The students use one-to-one correspondence to compare two sets of objects. Then they state whether one set has more, fewer, or as many members as the other set. Ordering of three or more sets is introduced at the kindergarten level; the counting of a small number of objects is also included.

In Grade 1, students acquire number concepts through counting objects, and they extend this knowledge into skip counting. Students also compare and order numbers. First graders progress into number concepts by using tens and ones to show place value. Teachers use models to introduce fraction recognition.

In Grade 2, students expand upon the number concepts of comparing, ordering, and skip counting. Comparing and ordering can include terms such as "greater than" and "less than" along with the symbols for these terms. Skip counting helps lead the child into investigating odd and even numbers. Students continue developing place value to the hundreds place. Recognition of fractional models is extended in Grade 2.

In Grade 3, students continue to develop the number concepts of comparing, ordering, and using odds, evens, and skip counting. Third graders begin to use place value and to identify decimal place value using money (tenths and hundredths). Using models, teachers introduce the students to the recognition of equivalent fractions. Reading and writing whole numbers begins in Grade 3.

In Grade 4, students are introduced to the rounding of whole numbers. Students extend their knowledge of decimals using place value. They read and write whole and decimal numbers and develop their recognition of equivalent fractions.

In Grade 5, students begin the use of exponential notation, and they use decimals in number operations. Equivalent fractions are identified without models.

In Grade 6, students study the history of numeration systems. They expand their rounding of numbers to include the rounding of decimals. Factoring, including prime and composite numbers, begins in this grade. Concepts and skills associated with an understanding of numbers include finding relationships between fractions, decimals, percents, and ratios. Students expand their knowledge of fractions by finding the least common multiple and the greatest common factor, and they are introduced to integers.

In Grade 7, students investigate the history of numeration systems. They are also introduced to the use of fractions, ratios, and percents. Seventh graders further develop their understanding of the rounding of decimals, factoring, and the use of integers; they expand their knowledge of finding the least common multiple and the greatest common factor.

In Grade 8, students use ratio and percent to develop their understanding of numbers. Scientific notation is introduced along with the process of determining significant digits. Students extend their knowledge of rounding, factoring, and using integers.

Essential Element 2: Operations

The second essential element addresses the basic operations on numbers (addition, subtraction, multiplication, and division), their properties, and their uses. These operations will be expanded upon at each grade level. The expansion may include numbers with more digits or fractions with different denominations.

Kindergarten students explore physical representations of addition and subtraction at the readiness level. In a story, the teacher can use objects to show the action of additive or subtractive movement.

In Grade 1, students acquire the number concepts of joining and separating sets using manipulatives. They use the manipulatives to find and write addition and subtraction equations. First graders compute and write the difference in subtraction equations and the sum in addition equations.

Second graders expand on the concept of addition and subtraction of whole numbers. In Grade 2, the teacher demonstrates the concept of multiplication of whole numbers by using concrete materials such as cubes arranged in rectangular arrays.

Grade 3 students are introduced to addition and subtraction of decimals using money. The students multiply two one-digit whole numbers. They begin division by using concrete objects. The meaning of division can be demonstrated by dividing a set of objects into equivalent sets. Students apply concepts of inverse operations with addition/subtraction and multiplication/division.

In Grade 4, students add and subtract decimals without using money notation. Students expand their understanding to the symbolic level of the division of whole numbers. Larger numbers are used in multiplication. Students use the inverse operations to find solutions to equations. Using models, they investigate the addition and subtraction of fractions with like denominators.

In Grade 5, the study of operations is advanced to addition, subtraction, and multiplication of decimals and fractions, not to include mixed numbers.

In Grade 6, students are introduced to addition, subtraction, multiplication, and division of integers, fractions, and decimals, teachers should stress understanding through the use of concrete materials.

Students in Grade 7 continue addition, subtraction, multiplication, and division of rational numbers. They are also introduced to the use of exponents.

Grade 8 students expand on the operations of rational numbers and their properties. Students investigate the process of finding the absolute value.

Essential Element 3: Problem Solving

Essential Element 3 addresses the process of selecting and matching problem-solving strategies to unfamiliar situations in which the method of solution and answer are not readily apparent. In K-Grade 8, students are taught a variety of increasingly sophisticated strategies for problem solving. At each grade level, students are expected to practice selecting the correct strategy using the types of problems being studied at that grade level.

In kindergarten, children begin to study problem solving through oral activities. They act out solutions using people and objects.

Through the year, first grade students progress from the study of concrete objects to abstract models. For example, they might begin working with interlocking counting cubes and move to drawing pictures to demonstrate their understanding of a problem. Students model operations for joining and separating as the teacher tells a story problem.

Second grade students apply their problem-solving skills to problems requiring addition and subtraction. They learn new strategies to use with problems that employ charts and graphs.

Third graders are expected to use estimation with whole numbers as a part of the problem-solving process. They continue to develop problem-solving strategies and to apply them to new situations. The new situations include multiplying with whole numbers and working with decimals using money, geometric models, and simple two-operation problems.

Fourth grade students solve more complex problems that require the use of the four basic operations with whole numbers, addition and subtraction with decimals, and addition and subtraction of fractions with like denominators only (using models). They extend their use of estimation to include decimals. Students are also taught problem-solving strategies for problems that require the use of two different operations.

In Grade 5, students continue to develop their problem-solving skills in all of the areas covered in Grade 4 and add the operations of multiplication of decimals and fractions excluding mixed numbers. They learn strategies to solve problems requiring three or more operations. At this level, problem-solving instruction focuses on the process used by the student to select a particular strategy. The student is asked to recognize a problem that has more than one solution, no solution, not enough information, or too much information. These mixed problem situations are an integral part of the problem-solving activities for Grades 5 through 8.

Sixth grade students are presented a wide variety of problem situations including basic operations on rational numbers; geometric concepts such as perimeter, area, and volume; and averages and probability. Teachers should encourage students to seek more than one method of solution and to discuss with the teacher and other students processes for choosing particular methods or strategies. Students are expected to write and solve simple linear equations.

In Grade 7, instruction emphasizes estimating the solution and adds to this skill new situations involving proportion, percent, and personal finance. Calculators and/or computers are used where appropriate to keep the focus on the process, not on the answer. Students extend their problem-solving skills to the use of inequalities for problems with more than one solution.

Eighth grade students use probability and statistics in problem-solving situations. They practice writing and solving equations. At the conclusion of Grade 8, the student should understand and be able to use the problem-solving process. Given an unfamiliar situation, the student should be able to (a) determine the nature of the solution, (b) select one or more appropriate strategies, (c) estimate the solution, (d) solve the problem, and (e) look back to determine if the solution fulfills the requirements of the problem.

Essential Element 4: Measurement

The fourth essential element addresses measurement concepts and skills using metric and customary units.

In kindergarten, children begin this study by identifying coins and units of time.

First grade students expand their study of time to include parts of an hour and their study of money to include paper money. The teacher introduces the concept of measuring length, weight/mass, liquid volume, and temperature by having the students use nonstandard measuring instruments.

Second grade students progress to the use of standard measuring instruments in both metric and customary units.

In the third grade, students use their understanding of measurement to solve problems involving metric and customary units and to find the perimeter of a figure when the lengths of the sides are known.

Fourth grade students solve word problems with metric and customary units. They are introduced to the conversion of metric units within the metric system and to the concept of area using manipulatives and grids. They extend their understanding of perimeter to regular and irregular polygons.

Fifth grade students apply metric and customary units to real-world problems. They find area using simple formulas, convert units within the metric system, and begin their study of volume using cubes.

Sixth grade students expand the use of metric and customary units in applications, perimeter problems, and circumference problems. Students progress from the concrete to the abstract level to find area, surface area, and volume.

In Grade 7, students apply these measurement skills to a variety of different problems including conversion within the metric system.

Eighth grade students use measurement concepts in problem-solving activities and applications. They learn to recognize precision and accuracy, and they apply estimation to solve problems.

Essential Element 5: Geometry

The fifth essential element addresses the properties and relationships of geometric shapes and their applications. At all grade levels, teachers are expected to use models and manipulatives to teach geometric concepts and their relationship to the physical world.

In kindergarten, students begin the study of geometry by classifying objects according to size and shape. Teachers introduce the concept of patterns using geometric shapes.

In Grade 1, students learn to recognize and name common two-dimensional shapes such as circles, squares, and triangles both in the environment and in models.

Second grade students use their recognition of two-dimensional models to differentiate shapes such as rectangles and parallelograms. Students learn to recognize and name models for common three-dimensional shapes.

Grade 3 students begin to identify the characteristics of common two- and three-dimensional shapes. This process includes investigating characteristics to decide how two shapes are alike and how they are different.

Grade 4 students investigate the properties of two- and three-dimensional figures. This study includes the process of developing accurate definitions. Students are introduced to the concepts of similarity, congruence, and symmetry.

Fifth grade students use the properties of two- and three-dimensional figures to recognize relationships, simplify models, and solve real-world problems. Students work with similarity, congruence, and symmetry with two- and three-dimensional figures. Teachers introduce the concept of surface area using models.

In Grade 6, students expand their abilities to use the properties of two- and three-dimensional figures in problem-solving situations. They learn to apply the concepts of similarity, congruence, and symmetry in problem situations. When they progress to the next level of understanding, students study translations, reflections, rotations, and their applications.

Seventh graders demonstrate their understanding of the properties of two- and three-dimensional figures through applications and the development of precise definitions. Students use points to define lines, rays, angles, and planes. They begin construction of geometric figures by paper folding and by compass and ruler.

Eighth grade students continue to apply the geometric concepts and skills learned in previous grades to increasingly more sophisticated problems. Construction of geometric figures is continued from Grade 7. Students' knowledge of properties of a right triangle is extended to include the Pythagorean property, similar triangles, and indirect measurement.

Essential Element 6: Graphing

The sixth essential element addresses the representation of numbers on a line, pairs of numbers on a line, and pairs of numbers on a coordinate plane. Students progress from a study of number lines with whole numbers to the study of coordinate planes.

Kindergarten children experience ordering points on a line at the readiness level. This study includes identifying directions and positions such as up, down, top, and bottom. First grade students begin to order numbers on a number line. In Grade 2, students develop the concept of locating points on a number line and are introduced to working with addition tables. In Grade 3, students extend this concept to working with multiplication tables. Fourth graders progress to drawing number lines to include fractions with like denominators and decimals. In Grade 5, students continue drawing number lines with fractions of unlike denominators and decimals.

Sixth graders begin graphing points on a coordinate plane. In Grade 7, graphing on a coordinate plane is extended to similar figures, reflection images, and linear equations. At this level, students begin applying skills they have acquired using a coordinate plane. In Grade 8, graphing on a coordinate plane is expanded to translations, reflections, and nonlinear graphs. Students also continue to apply skills they have acquired using the coordinate plane.

Essential Element 7: Probability and Statistics

The study of probability and statistics includes collecting and interpreting data. Kindergarten children experience classification and ordering objects at the readiness level. They order at least three objects according to length, height, capacity, and weight.

In Grade 1, students classify objects and order numbers. Second graders begin collecting and organizing information. They are introduced to reading bar graphs and making picture graphs. In Grade 3, students construct both bar and picture graphs.

Grade 4 students continue to expand their understanding of collecting data and using the data to construct graphs. They interpret pictographs and use this information to solve story problems. They also read and interpret line graphs. Students are introduced to listing the possible outcomes in a given situation.

Fifth grade students are introduced to interpreting charts and to using counting arrangements. By drawing different colored cubes from a box, students begin making predictions based upon a sample.

Students in Grade 6 advance to constructing their own sample spaces. The study includes determining and interpreting fractional probability. Finding averages is part of the students' work in statistics. Application of probability and statistics concepts are used to solve increasingly difficult problems.

In Grade 7, students use statistics to find the mean, median, and mode; they also use frequency distributions. The use of simple event probability is introduced. Students continue applying skills they have acquired with the use of probability and statistics.

In Grade 8, students use the tables and graphs they have interpreted to solve problems. They are introduced to the use of counting methods and factorials, making predictions, and drawing Venn diagrams.

Essential Element 8: Calculators and Computers

Essential Element 8 addresses the use of computing devices and their capabilities in the classroom. Although the use of calculators in the mathematics classroom is only called for specifically in seventh and eighth grade essential elements, teachers are encouraged to use calculators at all grade levels in mathematics classrooms. Calculators can be used effectively in several areas of the mathematics curriculum, especially in problem solving. Using a calculator can help a student keep the focus on the problem-solving process rather than on the computation process.

Grade 7 and 8 students use calculators and/or computers to solve a variety of mathematics problems.

In Grade 8, students learn the strengths and limitations of calculators and computers.

Essential Elements

Mathematics, Kindergarten

Essential elements	**Subelements**
Mathematics, kindergarten, shall include the following essential elements:	The student shall be provided opportunities to experience at the readiness level:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) number concepts (B) one-to-one correspondence (C) ordering (D) counting (objects)
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	physical representations of addition and subtraction
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) oral problem solving with objects and stories (B) identifying patterns
(4) Measurement concepts and skills using metric and customary units	(A) comparing (B) identifying money (C) identifying time units
(5) Properties and relationships of geometric shapes and their applications	(A) classification (B) size and shape (C) patterns
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	ordering points on a line
(7) The use of probability and statistics to collect and interpret data	(A) classification (B) ordering

Mathematics, Grade 1

Essential elements	**Subelements**
Mathematics, Grade 1, shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) acquire number concepts through (i) counting (objects) (ii) skip counting (iii) comparing and ordering (B) use tens and ones to show place value (C) recognize fractions (models)
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) acquire number operation concepts of joining and separating using manipulatives (B) add and subtract whole numbers
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) apply addition and subtraction (B) identify story situations depicting the joining and separating process (C) draw pictures depicting the joining and separating process (D) use modeling for presenting and solving problems
(4) Measurement concepts and skills using metric and customary units	(A) measure (using measuring instruments) (B) use measurement units for (i) money (ii) time (iii) length (iv) weight/mass (v) liquid volume (vi) temperature
(5) Properties and relationships of geometric shapes and their applications	recognize and name models of two-dimensional shapes
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	(A) order numbers (B) locate points on a number line
(7) The use of probability and statistics to collect and interpret data	(A) classify objects (B) order numbers

Mathematics, Grade 2

Essential elements	Subelements
Mathematics, Grade 2, shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) acquire number concepts through (i) comparing and ordering (ii) skip counting (iii) investigating odds and evens (B) use hundreds, tens, and ones to show place value (C) recognize fractions (models)
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) add and subtract whole numbers (B) demonstrate multiplication concepts of whole numbers (using concrete materials)
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) apply addition and subtraction (B) use charts and graphs
(4) Measurement concepts and skills using metric and customary units	(A) measure (using measuring instruments) (B) use measurement units of (i) money (ii) time (iii) length (iv) weight/mass (v) liquid volume (vi) temperature
(5) Properties and relationships of geometric shapes and their applications	recognize and name models of two- and three-dimensional shapes
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	(A) locate points on a number line (B) work with addition tables
(7) The use of probability and statistics to collect and interpret data	(A) collect and organize information (B) read bar graphs (C) make picture graphs

Mathematics, Grade 3

Essential elements	Subelements
Mathematics, Grade 3, shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) compare and order (B) use odds, evens, and skip counting (C) use place value (D) identify decimal place value using money (tenths and hundredths) (E) use fractions (models) (F) recognize equivalent fractions (using models) (G) read and write whole numbers
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) add and subtract whole numbers (B) add and subtract decimals using money (C) multiply whole numbers (D) divide whole numbers (using concrete materials) (E) recognize inverse operations
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate with whole numbers (B) use basic operations with whole numbers and with decimals using money (C) use geometry (D) use charts and graphs
(4) Measurement concepts and skills using metric and customary units	(A) measure (using measuring instruments) (B) solve problems using metric and customary unit measures (no conversion) (C) find perimeter
(5) Properties and relationships of geometric shapes and their applications	identify characteristics of two- and three-dimensional shapes
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	(A) locate points on a number line (B) work with addition and multiplication tables
(7) The use of probability and statistics to collect and interpret data	(A) collect and organize information (B) interpret and construct picture and bar graphs

Mathematics, Grade 4

Essential elements	**Subelements**
Mathematics, Grade 4, shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) compare and order (B) round numbers (C) use place value (D) use decimals (place value) (E) use fractions (models) (F) recognize equivalent fractions (using models) (G) read and write whole numbers and decimals
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) add and subtract whole numbers and decimals (B) multiply whole numbers (C) divide whole numbers (D) use inverse operations (E) add and subtract fractions using models (with like denominators)
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate with whole numbers and decimals (B) use basic operations with whole numbers, decimals, and fractions (C) use geometry (D) use charts and graphs
(4) Measurement concepts and skills using metric and customary units	(A) measure (using measuring instruments) (B) solve problems using metric and customary unit measures (no conversion) (C) find relationships between metric units (D) find perimeter (E) find area (with grids)
(5) Properties and relationships of geometric shapes and their applications	(A) investigate the properties of two- and three-dimensional figures (B) work with similarity, congruence, and symmetry
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	draw number lines with fractions (like denominators) and decimals

Mathematics, Grade 4, *continued*

Essential elements Mathematics, Grade 4, shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(7) The use of probability and statistics to collect and interpret data	(A) collect data and use to construct graphs (B) interpret pictographs and use information to solve story problems (C) list the possible outcomes in a given situation (D) read and interpret line graphs

Mathematics, Grade 5

Essential elements	**Subelements**
Mathematics, Grade 5, shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) compare and order (B) use exponential notation (C) round numbers (D) use place value (E) use decimals (F) use fractions (models) (G) identify equivalent fractions
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) add, subtract, multiply, and divide whole numbers (B) add, subtract, and multiply decimals and fractions (not including mixed numbers) (C) use inverse operations
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate with whole numbers and decimals (B) use basic operations with whole numbers, fractions, and decimals (C) apply geometric concepts (D) use charts and graphs
(4) Measurement concepts and skills using metric and customary units	(A) measure (using measuring instruments) (B) use metric and customary units in applications (no conversion between systems of measure) (C) convert units within the metric system (D) find perimeter and circumference (E) find area (with grids) (F) find volume (with cubes)
(5) Properties and relationships of geometric shapes and their applications	(A) use the properties of two- and three-dimensional figures (B) work with similarity, congruence, and symmetry (C) find surface area (with models)
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	draw number lines with fractions and decimals

Mathematics, Grade 5, *continued*

Essential elements	**Subelements**
Mathematics, Grade 5, shall include the following essential elements:	The student shall be provided opportunities to:
(7) The use of probability and statistics to collect and interpret data	(A) collect data and use to construct graphs (B) interpret and construct charts and graphs (C) use counting arrangements (D) predict possible outcomes from a sample

Mathematics, Grade 6

Essential elements Mathematics, Grade 6, shall include the following essential elements:	Subelements The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) study the history of numeration systems (B) round whole numbers and decimals (C) factor (primes and composites) (D) use place value (E) find relationships between fractions, decimals, percents, and ratio (F) find the least common multiple (G) find the greatest common factor (H) use integers (I) use exponential notation
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) add, subtract, multiply, and divide integers (B) use inverse operations (C) add, subtract, multiply, and divide fractions and decimals
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate with whole numbers and decimals (B) use basic operations with whole numbers, fractions, and decimals (C) solve simple linear equations (D) apply geometric concepts (E) use charts and graphs
(4) Measurement concepts and skills using metric and customary units	(A) measure (using measuring instruments) (B) use metric and customary units in applications (no conversion between systems of measure) (C) find perimeter and circumference (D) find area, surface area, and volume
(5) Properties and relationships of geometric shapes and their applications	(A) use the properties of two- and three-dimensional figures (B) apply similarity, congruence, and symmetry in problem situations (C) study translations, reflections, and rotations and their applications
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	graph points on a coordinate plane

Mathematics, Grade 6, *continued*

Essential elements	**Subelements**
Mathematics, Grade 6, shall include the following essential elements:	The student shall be provided opportunities to:
(7) The use of probability and statistics to collect and interpret data	(A) collect data and use to construct graphs (B) interpret and construct charts and graphs (C) construct sample spaces (D) determine and interpret fractional probability (E) find averages (F) apply probability and statistics concepts to solve problems

Mathematics, Grade 7 (1 unit)

Essential elements Mathematics, Grade 7, shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) investigate the history of numeration systems (B) round whole numbers and decimals (C) use scientific notation (D) factor (primes and composites) (E) find the least common multiple (F) find the greatest common factor (G) use fractions, decimals, ratio, and percent (H) use integers (I) use exponential notation
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) add, subtract, multiply, and divide rational numbers (B) use exponents
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate solutions (B) solve simple linear equations and inequalities (C) use basic operations (D) apply geometric concepts (E) use denominate numbers in problem solving (F) use proportion and percent (G) practice skills in personal finance
(4) Measurement concepts and skills using metric and customary units	(A) use metric and customary units (B) convert within the metric system (C) determine perimeter and circumference (D) find area, surface area, and volume (E) apply skills
(5) Properties and relationships of geometric shapes and their applications	(A) use points to define lines, rays, angles, and planes (B) understand properties and definitions of two- and three-dimensional figures (C) use similarity, congruence, and symmetry (D) construct geometric figures (E) apply skills

Mathematics, Grade 7, *continued*

Essential elements	**Subelements**
Mathematics, Grade 7, shall include the following essential elements:	The student shall be provided opportunities to:
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	(A) graph on a coordinate plane (i) similar figures (ii) reflection images (iii) linear equations (B) apply skills
(7) The use of probability and statistics to collect and interpret data	(A) use frequency distributions (B) find the mean, median, and mode (C) interpret and construct charts and graphs (D) use simple event probability (E) apply skills
(8) Investigating the use of computing devices and their capabilities	use calculators and/or computers to solve mathematics problems

Mathematics, Grade 8 (1 unit)

Essential elements	**Subelements**
Mathematics, Grade 8, shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with the understanding of numbers (whole, integer, and non-negative rational) and the place-value system	(A) use exponential notation (B) round whole numbers and decimals (C) determine significant digits (D) use scientific notation (E) factor (F) use ratio and percent (G) use integers
(2) The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	(A) demonstrate rational number properties (B) add, subtract, multiply, and divide rational numbers (C) find the absolute value
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate solutions (B) write and solve equations (C) solve problems using denominate numbers (D) apply geometric concepts (E) apply proportion and percent (F) use graphs (G) use probability and statistics (H) practice skills in personal finance
(4) Measurement concepts and skills using metric and customary units	(A) use metric and customary units (B) convert within the metric system (C) recognize precision (D) find area, surface area, and volume (E) apply skills
(5) Properties and relationships of geometric shapes and their applications	(A) use right triangle geometry with (i) Pythagorean property (ii) similarity (iii) indirect measurement (iv) ratios (B) apply skills (C) construct geometric figures
(6) The representation of numbers on a line and pairs of numbers on a coordinate plane	(A) graph on a coordinate plane (i) translations and reflections (ii) linear equations (iii) nonlinear graphs (B) apply skills

Mathematics, Grade 8, *continued*

Essential elements	**Subelements**
Mathematics, Grade 8, shall include the following essential elements:	The student shall be provided opportunities to:
(7) The use of probability and statistics to collect and interpret data	(A) use counting methods and factorials (B) draw Venn diagrams (C) find the probability of simple and compound events (D) use tables and graphs (E) find the mean, median and mode (F) make predictions (G) apply skills
(8) Investigating the use of computing devices and their capabilities	(A) use calculators and/or computers to solve mathematics problems (B) determine limitations and strengths

Fundamentals of Mathematics (1 unit)

Essential elements	**Subelements**
Fundamentals of Mathematics shall include the following essential elements:	The student shall be provided opportunities to:
(1) Measurement concepts and skills using metric and customary units	(A) find appropriate units of measure (B) solve problems involving measures (C) apply ratio and proportion
(2) Properties and relationships of shapes and their applications	(A) find and use perimeter, area, circumference, surface area, and volume (B) apply geometric properties
(3) Concepts and skills associated with the understanding of numbers (whole, integer, and rational) and the place-value system	(A) use the operations of addition, subtraction, multiplication, and division with whole numbers and integers in practical situations (B) use fractions and their relative values (C) use the operations of addition, subtraction, multiplication, and division with fractions in problem situations (D) compare decimal fractions (E) use the operations of addition, subtraction, multiplication, and division with decimals in problem situations
(4) Concepts and skills involving income, cost comparisons, and taxes	(A) compute wages (B) interpret deductions on paycheck stubs (C) apply percents (D) make budgets (E) open and maintain checking and savings accounts
(5) Experience in solving problems by selecting and matching strategies to given situations	(A) estimate solutions (B) write and solve equations (C) measure (D) apply geometric concepts (E) determine proportion and percent (F) use graphs (G) use probability and statistics (H) work with personal finance

Fundamentals of Mathematics, *continued*

Essential elements	**Subelements**
Fundamentals of Mathematics shall include the following essential elements:	The student shall be provided opportunities to:
(6) The use of probability and statistics to collect and interpret data	(A) use counting methods and factorials (B) construct charts, graphs, and frequency tables (C) find measures of central tendency (mode, median, mean) (D) understand simple and compound events (E) make predictions (F) apply skills

Consumer Mathematics (1 unit)

Essential elements	**Subelements**
Consumer Mathematics shall include the following essential elements:	The student shall be provided opportunities to:
(1) Use of concepts and skills in graphing and in statistics to gather information and make decisions	(A) interpret charts, tables, graphs, and maps (B) construct appropriate graphs from consumer or career-related information (C) determine wages and deductions from given information (D) use mean, median, and mode (E) find validity of claims involving statistical data
(2) Application of operations and properties of ratio, proportion, and percent to determine costs and to make decisions from cost comparisons	(A) find costs of (i) housing (owning and renting) (ii) food (iii) transportation (iv) clothing (v) insurance (B) make decisions by comparing different types of the costs in subparagraph (A) of this paragraph (C) find discounts, sales, rebates, coupons, etc.
(3) Use of probability concepts and skills to investigate possibilities of arrangements or occurrences and to make predictions from these probabilities	(A) use the fundamental counting principle (B) use permutations (C) construct tree diagrams (D) use sample spaces to determine probabilities and odds in favor of an event
(4) Investigation of financing procedures and costs, and comparison of different types of financing	(A) investigate the use of credit cards and charge accounts (B) study options of loans for things such as car, home, etc.
(5) Concepts and skills involved in banking and investment procedures	(A) use banking forms (deposit slips, checks, check book records, bank statements) (B) compare advantages and disadvantages of investments
(6) Concepts and skills involved in following local, state, and federal tax laws	(A) investigate local, state, and federal tax requirements (B) find various taxes on specified amounts (C) use tax reporting forms

Consumer Mathematics, *continued*

Essential elements	**Subelements**
Consumer Mathematics shall include the following essential elements:	The student shall be provided opportunities to:
(7) Concepts and skills involved in developing a budget	(A) use the knowledge of costs and cost comparisons to make plans for managing specific amounts of money (B) develop a personal budget

Pre-Algebra (1 unit)

Essential elements	**Subelements**
Pre-Algebra shall include the following essential elements:	
(1) Concepts and skills associated with the understanding of numbers and the place-value system	(A) find whole number and decimal place values (B) use scientific notation (C) find factors and multiples (D) understand fraction concepts involving (i) least common multiple (ii) greatest common factor (E) use integers
(2) Operations on numbers, their properties, and their uses	(A) understand basic operations with and properties of integers, decimals, and fractions (B) use the order of operations (C) use exponents (D) find squares and square roots (E) solve simple equations involving integers, decimals, and fractions (F) apply skills
(3) Experience in solving problems by selecting and matching strategies to given situations	(A) evaluate variables and expressions (formulas) (B) use ratio and proportion (C) find percent and percentage (D) determine needs related to personal finance
(4) Investigation of the properties and relationships of geometric figures using measurement concepts and skills	(A) understand the basic elements of geometry (point, line, etc.) (B) investigate geometric figures and their characteristics (C) use metric and customary units (D) find distance, perimeter, circumference, area, surface area, and volume (E) use indirect measurement with similar triangles (F) investigate right triangle properties

Pre-Algebra, *continued*

Essential elements	**Subelements**
Pre-Algebra shall include the following essential elements:	The student shall be provided opportunities to:
(5) The use of probability and statistics to collect and interpret data	(A) use counting procedures (tree diagrams, multiplication) (B) find the probability of independent and dependent events (C) gather and record data (D) determine the mean, median, mode (E) construct graphs from gathered data (F) interpret graphs and analyze data
(6) Concepts and skills involved in the analysis and graphing of relations and functions	(A) graph solution sets of equations and inequalities on a line (B) use ordered pairs and coordinate graphing (C) simplify polynomials (D) investigate linear equations and inequalities and their graphs (E) apply direct and inverse variation (F) find slope and intercepts (G) solve systems of equations (H) apply skills

Informal Geometry (1 unit)

Essential elements	**Subelements**
Informal Geometry shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills involving measurement	(A) develop the concept of measurement using measuring instruments (metric and customary) (B) estimate measures (C) choose appropriate units of measurement
(2) Two- and three-dimensional geometric figures and shapes	(A) recognize two- and three-dimensional figures by name (B) demonstrate an understanding of the properties of plane and solid figures (C) use the properties to solve problems (D) recognize various triangles by shape and use their associated line segments such as altitudes, angle bisectors, and perpendicular bisectors of sides
(3) Parallelism and perpendicularity	(A) recognize and use parallel and perpendicular lines with associated facts regarding the angles formed (B) recognize and use key facts (standard theorems) about common quadrilaterals such as rectangles, parallelograms, rhombuses, squares, and trapezoids
(4) Concepts of congruence and similarity including scale drawing	(A) demonstrate an understanding of congruence as applied to line segments, angles, and geometric figures (B) demonstrate a knowledge of similarity and similar figures (C) use ratios and proportions to solve for missing parts (D) apply similarity to maps and scale drawings

Informal Geometry, *continued*

Essential elements Informal Geometry shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(5) Constructions via paper folding as well as compass and straightedge	(A) make simple constructions using paper folding and compass and straightedge (i) copy a line segment (ii) copy an angle (iii) bisect an angle (iv) construct the perpendicular to a line from a given point outside a line and from a point on a line (v) construct a line parallel to a given line (vi) divide a given segment into a specified number of congruent segments (B) use paper folding to construct regular polygons (C) use compass and straightedge to construct regular polygons
(6) Basic coordinate geometry	(A) use Cartesian coordinates (B) find the midpoint and the slope of a line (C) find the distance between points (D) use locus to describe geometric figures (E) translate points, lines, and geometric figures in the plane
(7) Area and perimeter	(A) find areas and perimeters of triangles, parallelograms, trapezoids, and regular polygons (B) develop the respective formulas intuitively or with models (C) apply the formulas to practical problems involving linear and square units (D) find the approximate areas and perimeters of irregularly shaped figures
(8) The right triangle	(A) use the Pythagorean Theorem in applications to right triangles (B) recognize and use trigonometric ratios of sine, cosine, and tangent as related to the right triangle (C) solve practical problems involving the trigonometric ratios

Informal Geometry, *continued*

Essential elements Informal Geometry shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(9) The circle and sphere	(A) recognize and use circles and spheres with related parts such as radius, diameter, arc, chord, tangent, secant, and sector (B) compute circumferences and areas of circles (C) solve practical problems involving measurements of circles and spheres
(10) Volume and surface area	(A) compute the lateral and surface area of common solids (B) apply the formulas to practical problems related to areas (C) compute the volume of common solids (D) apply the formulas to practical problems related to capacity and weight (E) find the surface area or volume of irregularly shaped figures

Algebra I (1 unit)

Essential elements	Subelements
Algebra I shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills involving operations with real numbers and properties of real numbers	(A) classify real numbers as integral, rational, or irrational (B) define and use the properties of the real numbers (C) graph real numbers (D) use the order of operations including addition, subtraction, multiplication, and division (E) find absolute value, exponents, additive and multiplicative inverses
(2) Linear equations and inequalities in one variable	(A) solve equations (B) solve inequalities (C) graph equations and inequalities (D) solve equations and inequalities involving absolute value (E) use equations and inequalitites in applications and problem-solving situations (F) use formulas to solve problems
(3) Linear equations and inequalities in two variables	(A) solve equations and formulas in two variables (B) find intercepts and slope to graph equations (C) identify and graph functions (D) find the equation of a line (E) use the graphing, addition, and substitution methods of solving a system of equations (F) graph systems of inequalities (G) use equations and inequalities in applications and problem-solving situations
(4) Concepts and skills associated with polynomials	(A) add, subtract, multiply, and divide polynomials (B) factor polynomials
(5) Concepts and skills associated with rational expressions	(A) simplify rational expressions (B) solve problems using ratio and proportion (C) add, subtract, multiply, and divide rational expressions

Algebra I, *continued*

Essential elements Algebra I shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(6) Properties of roots and operations with radicals	(A) find square roots (B) add, subtract, multiply, and divide square roots (C) simplify radicals (D) solve simple radical equations
(7) Concepts and skills associated with quadratic equations	(A) graph quadratic equations and functions (B) solve quadratic equations by using square roots, graphing, factoring, completing the square, and using the quadratic formula (C) apply quadratic equations

Geometry (1 unit)

Essential elements Geometry shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(1) The nature of deductive reasoning.	(A) draw conclusions (B) use logic (C) use undefined terms and definitions (D) use direct proof (E) use indirect proof
(2) Geometry of the real world	(A) find properties of geometric figures (B) understand symmetry (C) find area and volume
(3) Fundamental ideas: lines and angles	(A) determine distance and betweenness (B) measure angles
(4) Some basic postulates and theorems	(A) investigate postulates of equality (B) use bisection theorems (C) understand angle relationships (D) use right angle theorems (E) prove theorems
(5) Congruent triangles	(A) use congruence postulates (B) prove triangle congruence (C) perform straightedge and compass constructions
(6) Transformations	study reflections, translations, and rotations
(7) Inequalities	(A) use postulates of inequality (B) use the exterior angle theorem (C) use triangle side and angle inequalities (D) use the triangle inequality theorem
(8) Parallel lines	(A) find parallel and perpendicular lines (B) use the parallel postulate (C) identify angles related to parallel lines
(9) Quadrilaterals	(A) understand classifications and definitions (B) use properties of the parallelogram, rhombus, rectangle, square, and trapezoid (C) prove theorems

Geometry, *continued*

Essential elements Geometry shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(10) Area	(A) find the area of polygonal regions (B) apply Heron's Theorem
(11) Similarity	(A) use ratio and proportion (B) work with similar triangles (C) find proportional line segments (D) use the angle bisector theorem (E) find perimeters of similar triangles
(12) The right angle	(A) understand proportions of right triangles (B) use the Pythagorean Theorem (C) investigate special right triangles (30°, 60°, isosceles)
(13) Coordinate geometry	(A) use Cartesian coordinates (B) define and apply locus theorems (C) define the straight line (D) graph the straight line (E) find distances
(14) Circles	(A) investigate segments related to circles (B) understand angles of a circle (C) investigate inscribed and circumscribed circles
(15) Regular polygons and the circle	(A) use related theorems (B) find perimeter of regular polygons (C) find area of regular polygons (D) find circumference and area of a circle (E) investigate sectors and arcs
(16) Geometric solids	(A) investigate lines and planes in space (B) investigate polyhedrons, prisms, and spheres (C) find surface area and volume of prisms and cylinders (D) find surface area and volume of pyramids, cones, and spheres

Algebra II (1 unit)

Essential elements	**Subelements**
Algebra II shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with open sentences	(A) solve open sentences in one variable (B) solve systems of linear open sentences
(2) Properties of relations and functions	(A) investigate relations and their graphs (B) find inverse relations (C) investigate properties of functions (D) find the domain and range of a function (E) perform composition of functions (F) use direct and inverse variation
(3) Properties of the complex number system	(A) define a complex number, its additive inverse, conjugate, and absolute value (B) add, subtract, multiply, and divide complex numbers
(4) Concepts and skills associated with polynomials and rational expressions	(A) factor expressions over the set of real numbers (B) simplify rational expressions (C) perform operations of addition, subtraction, multiplication, and division involving rational expressions (D) simplify complex fractions
(5) Properties of points and planes in space	(A) use coordinate geometry (B) find the distance between points in a plane and find the midpoints of segments
(6) Concepts and skills associated with matrices and determinants	(A) add and multiply matrices (B) solve systems of equations using determinants and matrices (C) use determinants or matrices to solve problem situations
(7) Concepts and skills associated with quadratic functions	(A) develop and use the quadratic formula (B) find discriminants and zeros of quadratic functions (C) write quadratic equations from roots (D) draw graphs using the vertex and axis of symmetry

Essential elements Algebra II shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(8) Concepts and skills relating to conic sections and systems of quadratics	(A) graph equations of circles, ellipses, parabolas, and hyperbolas (B) write the equation of a parabola and a circle from the definitions (C) identify conic equations (D) graph quadratic systems and indicate solution sets (E) solve systems algebraically
(9) Concepts and skills relating to exponential and logarithmic functions	(A) graph functions and inverses (B) change equations from logarithmic to exponential form and conversely (C) solve logarithmic and exponential equations (D) solve problems using logarithmic and exponential equations
(10) Concepts and skills relating to higher degree polynomial functions	(A) use synthetic division (B) apply the Fundamental Theorem of Algebra (C) use the Rational Root Theorem (D) graph higher degree polynomial functions with real roots
(11) Concepts and properties of sequences and series	(A) investigate arithmetic and geometric sequences (B) find arithmetic and geometric means (C) investigate arithmetic and geometric series (D) investigate convergent geometric series (E) use sequences and series as models of problem situations (F) use factorials (G) expand binomial powers (H) use the Binomial Theorem

Trigonometry (½ unit)

Essential elements	**Subelements**
Trigonometry shall include the following essential elements:	The student shall be provided opportunities to:
(1) Trigonometric functions	(A) understand relations and functions (B) define the sine, cosine, tangent, secant, cosecant, and cotangent functions (C) find values of the trigonometric functions (D) solve right triangle problems
(2) Properties of trigonometric and circular functions	(A) graph functions (B) use radian measure (C) use periodic motion
(3) Applications of trigonometric and circular functions	(A) understand the reciprocal, the quotient, and the Pythagorean properties (B) use trigonometric identities (C) solve trigonometric equations
(4) Inverses of trigonometric and circular functions	(A) graph inverse functions (B) investigate properties of inverse functions
(5) Triangle problems	(A) use the law of cosines (B) use the law of sines (C) use vectors
(6) Application of trigonometric and circular functions	(A) use polar coordinates (B) use complex numbers (C) use power series (D) use hyperbolic functions

Elementary Analysis (½ unit)

Essential elements	**Subelements**
Elementary Analysis shall include the following essential elements:	The student shall be provided opportunities to:
(1) Properties of the real number system and other mathematical systems	(A) understand the structure of mathematical systems (undefined and defined terms, operations, postulates, and theorems) (B) investigate groups, rings, and fields (C) understand matrices (addition and multiplication) (D) understand ordered fields (properties, proofs)
(2) Concepts and skills involved in the analysis of relations and functions and their properties and graphs	(A) review domain, range, and inverse (B) find composition of functions (C) find inverse functions (D) identify increasing and decreasing functions (E) identify and graph periodic functions (F) perform operations on functions (G) identify continuous functions
(3) Properties and graphs of special functions	(A) use polynomial functions (B) use exponential functions (C) use logarithmic functions (D) use circular functions
(4) Logic concepts applied in mathematical induction	(A) find patterns and make generalizations (B) use the theorem of mathematical induction
(5) Concepts and skills related to higher degree polynomial functions	(A) use the Division Theorem and Remainder Theorem (B) use synthetic division (C) use the Rational Root Theorem (D) use Descartes' Rule of Signs and the Upper and Lower Bound Theorem (E) find rational roots of polynomials (F) apply the Fundamental Theorem of Algebra (G) graph polynomial functions
(6) Concepts and skills involved in working with sequences and series	(A) use general terms (B) perform summation proofs (C) use the Binomial Theorem, including factorials and combinations (D) find limits of sequences

Analytic Geometry (½ unit)

Essential elements Analytic Geometry shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(1) The plane	(A) investigate the geometry of ordered pairs (B) investigate lines, polygons, and curves (C) perform basic operations with vectors (D) use parametric equations
(2) Space	(A) use space coordinatization (B) investigate the geometry of ordered triples (C) investigate lines and planes (D) use vectors in space
(3) Curves in the plane	(A) use vector equations for lines (B) find Cartesian equations for lines (C) use families of lines (D) investigate the polar coordinate system (E) relate polar and Cartesian coordinates and equations (F) sketch and find the intersections of curves
(4) Circles, cylinders, and spheres	investigate equations and properties
(5) Conics	(A) derive the equations of circles, parabolas, ellipses, and hyperbolas (B) solve second-degree equations (C) use conics in applications
(6) Transformations	(A) use translations, invariants, reflections, rotations, and composition of transformations (B) use transformations to simplify equations

Precalculus (½-1 unit)

Essential elements	**Subelements**
Precalculus shall include the following essential elements:	The student shall be provided opportunities to:
(1) Real numbers and coordinates	(A) understand the real number line (B) use field properties (C) write coordinates in two- and three-space
(2) Functions and their graphs	(A) understand functions: notation, language, and graphs (B) perform operations on functions (C) identify decreasing, increasing, and periodic functions (D) graph functions (E) find composite and inverse functions
(3) Polynomial and rational functions	(A) use the Division Theorem and the Factor Theorem (B) use synthetic division (C) use the Rational Root Theorem (D) use Descartes' Rule of Signs and the Upper and Lower Bounds Theorem (E) find rational roots of polynomials (F) apply the Fundamental Theorem of Algebra (G) graph polynomial functions
(4) Exponential and logarithmic functions	(A) define exponential functions (B) apply exponential functions (C) define logarithmic functions (D) use properties and graph logarithmic functions (E) solve exponential and logarithmic equations (F) use natural logarithms (the number "e")
(5) Circular functions, their properties and applications	(A) find angle measurement (B) identify the wrapping function (C) identify periodic functions (D) use applications of circular functions (E) find inverse functions

Precalculus, *continued*

Essential elements Precalculus shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(6) Trigonometric functions, their properties and applications	(A) define the trigonometric functions (B) find the value of the trigonometric functions (C) graph the trigonometric functions (D) solve right triangle problems (E) solve general triangles (F) solve trigonometric equations (G) derive fundamental identities (H) prove and use identities
(7) Vectors	(A) investigate vectors in the plane (B) use vectors in space (C) use vector equations for lines
(8) Complex numbers	(A) find sums, products, differences, and quotients of complex numbers (B) represent complex numbers graphically (C) use the trigonometric forms of complex numbers (D) use polar coordinates (E) graph equations in polar form
(9) Sequences and series	(A) identify arithmetic and geometric sequences and series (B) find sequences and sums (C) use mathematical induction (D) find the limit of a sequence (E) use the Binomial Theorem
(10) Second degree relations	(A) derive the equations for circles, parabolas, ellipses, and hyperbolas (B) graph second degree equations (C) perform translation and rotation of axes (D) use conics in applications (E) solve general second degree equations

Calculus (½-1 unit)

Essential elements	**Subelements**
Calculus shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts associated with the limit of a function	(A) represent limits through geometric interpretations (B) apply various limit theorems (C) understand continuity
(2) Concepts and skills associated with the derivative	(A) investigate lines tangent to curves (B) derive the formulas for the derivatives from the limit of a function (C) use theorems and properties of the derivative (D) find maxima and minima (E) apply the Mean Value Theorem (F) sketch curves (G) use concepts and skills in applications (velocity, acceleration)
(3) Concepts and skills associated with the integral and techniques of integration	(A) approximate areas (B) describe areas as a limit (C) use the Fundamental Theorem of Calculus (D) develop and use formulas (E) find volumes of revolution by the disk and shell methods (F) work with continuous functions (G) integrate by substitution (H) integrate by parts (I) apply integral concepts to find fluid pressure, center of mass, etc.
(4) Applications of calculus to special functions	(A) differentiate and integrate special functions such as (i) trigonometic functions (ii) logarithmic functions (iii) exponential functions (iv) functions written in polar coordinates (B) apply concepts to differential equations
(5) Concepts and skills associated with infinite series	(A) relate infinite series to integrals (B) investigate properties of infinite series (C) test for convergence or divergence

Advanced Mathematics for Business (½-1 unit)

Essential elements	**Subelements**
Advanced Mathematics for Business shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills relating to probability and statistics	(A) construct sample spaces (B) apply counting principles (C) find the probability of simple and compound events (D) use the probability function (E) determine conditional probability (F) use Bayes' formula (G) construct frequency distributions (H) apply concepts to grouped data (I) determine mean, median, and mode (J) find and interpret variance and standard deviation
(2) Concepts relating to random variables and probability functions	(A) recognize random variables (B) apply probability functions for a finite random variable (C) identify Bernoulli random variables (D) describe continuous probability distributions (E) recognize and interpret standard normal distributions (F) find the expected value of a random variable (G) determine variance and standard deviation (H) use Chebyshev's inequality
(3) Concepts and skills associated with linear equations and inequalities	(A) study properties of linear equations and inequalities in one and two variables (B) work with a rectangular coordinate system (C) solve systems of linear equations and inequalities (D) work with matrices
(4) Concepts relating to linear programming	(A) use the geometric approach (B) apply the simplex method
(5) Concepts and skills relating to finance	(A) determine arithmetic and geometric means (B) find simple and compound interest (C) determine present and future values of annuities

Advanced Mathematics for Business, *continued*

Essential elements	**Subelements**
Advanced Mathematics for Business shall include the following essential elements:	The student shall be provided opportunities to:
(6) Concepts relating to limits	(A) interpret the definition geometrically (B) work with basic theorems (C) determine limits of simple polynomial expressions (D) find limits of functions requiring a "rationalizing" step
(7) Concepts and skills associated with derivatives	(A) interpret the definition geometrically (B) work with basic theorems and their applications (C) use the chain rule
(8) Concepts and skills associated with integration	(A) determine area by approximation (B) use summation notation and common theorems (C) show area as a limit (D) use the definite integral (E) describe area as an integral (F) apply basic theorems of integration

Mathematics of Consumer Economics (½-1 unit)

Essential elements	**Subelements**
Mathematics of Consumer Economics shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills used when evaluating personal income and benefits	(A) understand aspects of the free enterprise system and its benefits (B) investigate wages and deductions (C) investigate benefits offered by companies, unions, and organizations (D) investigate social security and income tax (E) investigate stocks and bonds
(2) Concepts and skills involved in banking, saving, and borrowing	(A) understand credit financing (B) investigate the Federal Reserve System (C) investigate local bank services (D) investigate trust funds and estate planning (E) investigate different types of loans (car, home, personal) (F) investigate types of institutions that offer loans
(3) Concepts and skills associated with personal expenditures	investigate expenditures for: (A) food, shelter, and clothing (B) transportation (C) insurance (D) medical and dental expenses (E) education (F) retirement and annuities (G) miscellaneous
(4) Concepts and skills involved in making a budget	(A) make informed consumer decisions (B) practice sound money management (spending and saving)

Computer Mathematics I (½-1 unit)

Essential elements	**Subelements**
Computer Mathematics I shall include the following essential elements:	The student shall be provided opportunities to:
(1) Fundamentals of computer systems	(A) investigate major components, including input, storage, processing, and output (B) learn computer-system vocabulary (C) investigate the binary system
(2) Programming in the BASIC language	(A) use flowcharts (B) learn elements of BASIC (C) interpret error messages (D) use system commands (E) interpret input and output (F) understand transfer of control (G) use FOR-NEXT loops (H) understand arrays, functions, and subroutines (I) use string variables (J) use BASIC in problem-solving applications
(3) Using the computer for topics in consumer mathematics	use the computer to solve problems related to: (A) salary, commission, and benefits (B) net pay (C) checking and savings accounts (D) ratio, proportion, and percent (E) loans and charge accounts (F) budgeting (G) investments
(4) Using the computer for topics in algebra	use the computer to solve problems related to: (A) absolute value (B) slope of the graph of a linear equation (C) x- and y- intercepts of the graph of a linear equation (D) graphs of functions and relations (linear, quadratic, polynomial) (E) solutions to linear equations and inequalities (F) nth roots of numbers (G) arithmetic and geometric means (H) factorization of polynomials (I) solutions of quadratic equations (J) solutions of quadratic systems

Computer Mathematics I, *continued*

Essential elements Computer Mathematics I shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(5) Using the computer for topics in geometry	use the computer to solve problems related to: (A) perimeter of quadrilaterals and triangles (B) area of rectangles, circles, triangles, trapezoids, and parallelograms (C) classification of angles as acute, right, or obtuse (D) volume of a rectangular solid, sphere, right circular cylinder, and prism (E) similar and congruent triangles (F) classification of triangles as equilateral, isosceles, or scalene (G) lateral area and total surface area of a right circular cylinder, and surface area of a sphere (H) missing sides of right triangles (I) collinear points in a coordinate plane (J) interior angles of a polygon (K) regular and nonregular polygons (L) segments and points dividing segments (midpoint, etc.)
(6) Using the computer for topics in number theory	use the computer to solve problems related to: (A) prime numbers and relatively prime integers (B) perfect, abundant, and deficient numbers (C) greatest common divisor and least common multiple (D) sorting numbers (E) divisibility (F) integral factorization
(7) Using the computer for topics in probability and statistics	use the computer to solve problems related to: (A) permutations and combinations (B) random numbers (C) mean, median, and mode

Computer Mathematics II (½-1 unit)

Essential elements	Subelements
Computer Mathematics II shall include the following essential elements:	The student shall be provided opportunities to:
(1) Computer structure and design	(A) understand internal organization of microcomputers (B) use peripherals (C) understand computer networking
(2) Numeration systems and alphanumeric codes	(A) learn the relationship between numeration systems and computing (B) understand binary, octal, hexadecimal numeration (C) perform conversions
(3) Programming languages and procedures	(A) understand syntactical requirements (B) learn elements of languages such as BASIC, COBOL, FORTRAN, Pascal, etc. (C) write structured programs (D) interpret file structures (E) use data storage and retrieval (F) perform manipulation of data (G) write sort and search routines (H) interpret formatted output (I) write CAI programs
(4) Using the computer for topics in algebra	use the computer to solve problems related to: (A) polynomial equations with rational roots (B) n x n systems of equations (C) the binomial theorem (D) sequences and series (E) matrix equations
(5) Using the computer for topics in coordinate geometry	use the computer to solve problems related to: (A) translations, rotations, and reflections (B) dilations and contractions (C) families of lines and curves (D) three-dimensional coordinate geometry (E) estimating the value of π
(6) Using the computer for topics in probability and statistics	use the computer to solve problems related to: (A) permutations (B) combinations (C) probability experiments—predicting (D) graphing data (E) random numbers

Essential elements	**Subelements**
Computer Mathematics II shall include the following essential elements:	The student shall be provided opportunities to:
(7) Using the computer for topics in advanced mathematics	use the computer to solve problems related to: (A) number theory (B) intuitive calculus (C) solving triangles by trigonometry (D) vectors
(8) Using the computer for topics in other content areas	use the computer to solve problems related to: (A) topics in science (B) topics in economics (C) topics in business

Probability and Statistics (½ unit)

Essential elements	**Subelements**
Probability and Statistics shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills associated with various counting procedures	(A) find permutations (B) count combinations (C) apply the Binomial Theorem
(2) Concepts and skills associated with determining the probability of an event	(A) perform probability experiments (B) identify independent and dependent events and their probabilities (C) construct sample spaces (D) investigate randomness in sampling (E) determine conditional probability (F) apply other probability rules (G) predict outcomes
(3) Experience with different types of probability distributions	(A) construct and interpret frequency distributions (B) identify random variables (C) determine arithmetic and geometric means (D) find standard deviation (E) apply Chebyshev's Theorem (F) identify continuous and joint distributions (G) recognize and interpret the normal curve (H) use the binomial distribution to find probabilities
(4) Use of inferential statistics to make decisions or to determine validity	(A) apply the theory of sampling (B) test hypotheses

History of Mathematics (an independent study course—½ unit)

Essential elements History of Mathematics shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(1) Historical development of numbers and numerals	(A) examine at least two ancient numeration systems (B) investigate place-value systems other than ten (C) research the origin of zero (D) investigate prime and composite numbers and persons associated with their development (E) understand rational numbers in fractional and decimal forms (F) research persons responsible for proof of the Pythagorean Theorem (G) investigate irrational numbers and the persons who researched them (H) investigate complex numbers and their development
(2) Historical development of methods of computation	(A) investigate calculating devices (B) examine the capability of various computers (C) investigate the use of logarithms
(3) Historical development of geometry	(A) examine the applications of practical geometry in ancient civilizations (B) investigate Euclidean and non-Euclidean geometries (C) investigate the golden rectangle and applications (D) examine geometry in construction techniques (E) research unsolvable construction problems from antiquity (F) study regular polyhedrons and their discoverers (G) examine conic sections

History of Mathematics, *continued*

Essential elements	**Subelements**
History of Mathematics shall include the following essential elements:	The student shall be provided opportunities to:
(4) Historical development of algebra	(A) contrast algebraic symbols with other symbols meaning the same thing, and research their use (B) examine the algebra of ancient civilizations (C) identify the contribution of Diophantus of Alexandria (D) investigate persons contributing to the solution of general polynomial equations of the 3rd, 4th, and 5th degrees
(5) Persons associated with modern mathematics	examine nineteenth or twentieth century mathematicians or both and their contributions

Survey of Mathematics (an independent study course—½ unit)

Essential elements Survey of Mathematics shall include the following essential elements:	**Subelements** The student shall be provided opportunities to:
(1) Concepts and skills involving sets and logic	(A) investigate relations, equivalence relations, and comparisons of finite and infinite sets (B) use truth tables (C) use binary variables to solve logic problems
(2) Concepts relating to numbers and numerals	(A) examine ancient numeration systems (B) apply properties and relationships of subsets of the complex number system
(3) Concepts relating to probability and statistics	(A) use patterns to establish conclusions (B) use Pascal's triangle (C) determine probability of events (D) find measures of central tendency (E) apply the normal distribution curve
(4) Concepts relating to mathematical systems	(A) explore finite and abstract systems and their operations (B) examine properties of groups and fields
(5) Concepts relating to numerical trigonometry	(A) solve practical problems involving right triangles using trigonometric ratios (B) solve practical problems involving oblique triangles using the laws of sines and cosines

Number Theory (an independent study course—½ unit)

Essential elements	**Subelements**
Number Theory shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts and skills involving the use of integers	(A) understand mathematical induction (B) apply the Sieve of Eratosthenes (C) use the Euclidean algorithm (D) apply the Fundamental Theorem of Arithmetic (E) find the number of divisors and the sum of divisors of a given integer (F) find the greatest common divisor (G) identify perfect numbers
(2) Concepts and skills relating to primes	(A) identify Pythagorean triples (B) prove there are infinitely many primes (C) investigate Mersenne numbers (D) find the Euler function of a given integer
(3) Concepts and skills associated with divisibility and congruence	(A) apply the properties of divisibility (B) identify field properties for given modular systems (C) determine a residue system for a given modulus (D) apply Euler's theorem (E) identify Diophantine equations (F) apply Wilson's theorem (G) apply the Chinese remainder theorem (H) find quadratic congruence

Linear Algebra (an independent study course—½ unit)

Essential elements	**Subelements**
Linear Algebra shall include the following essential elements:	The student shall be provided opportunities to:
(1) Linear equations and matrices	(A) solve linear systems using matrices (B) apply properties of matrix operations (C) identify the inverse of a matrix
(2) Determinants	(A) apply definitions and properties (B) use cofactor expansion (C) apply skills
(3) Vectors and vector spaces	(A) investigate (i) vectors in the plane (ii) n-vectors (iii) vector spaces and subspaces (B) identify linear independence (C) find the rank of a matrix (D) work with orthonormal bases in R^n
(4) Linear transformations	use linear transformation and matrix representations
(5) Eigenvalues, Eigenvectors	(A) investigate diagonalization (B) investigate diagonalization of symmetric matrices
(6) Applications	(A) apply skills to conic sections and quadric surfaces (B) apply skills to the theory of games (C) apply skills to the Fibonacci sequence

Linear Programming (an independent study course—½ unit)

Essential elements	**Subelements**
Linear Programming shall include the following essential elements:	The student shall be provided opportunities to:
(1) Concepts involving mathematical modeling	(A) investigate the objective function, the constraints, and the non-negativity requirement for a given linear programming model (B) graph the constraints, identifying convex and extreme points, the area of feasible solutions, and values of the variables that optimize
(2) Concepts and skills involving the simplex method	(A) solve a linear programming model by the simplex method (B) use a linear programming production model (C) set up linear programming models from problem situations (D) interpret final tableaus for optimal strategies and limiting factors
(3) Variations and applications of the simplex method	(A) use segmentation, multiple tableaus, or approximation by a linear function (B) use goal programming (C) use linear programming to find solutions to practical situational problems

Correlation of Essential Elements to TEAMS Objectives

Students' mastery of a portion of the essential elements is measured by the Texas Educational Assessment of Minimum Skills (TEAMS) tests at Grades 1, 3, 5, 7, and 9 and at the exit level. The tests are indicators of statewide progress on certain key objectives for mathematics. However, TEAMS covers only a small subset of the total mathematics program, and results should not be used as sole indicators of a quality program.

Skill areas and objectives measured by the TEAMS Test for 1986-90 are listed below. Following each objective, grade levels and/or subjects are listed with essential elements that address the specific TEAMS objective. This correlation indicates how the Chapter 75 curriculum can be used to prepare students for each TEAMS objective.

Grade 1*

Skill Area: Number Concepts

- Objective 1: Identify the correct solution to a problem involving number sequence.

Prekindergarten:	Cognition Development 1A, B, C/2A, B, C/3A/4A, B,C
Kindergarten:	1A, B, C, D
Grade 1:	1A, 1B, 6A

- Objective 2: Identify the place value for a given problem.

Grade 1:	1A, B

- Objective 3: Identify the correct solution to a problem involving number comparison.

Kindergarten:	1A, B, C, D
Grade 1:	1A, B

Skill Area: Computation

- Objective 4: Add two whole numbers whose sum is less than 20.

Prekindergarten:	Cognition Development 2D, 3B, 4C
Kindergarten:	2
Grade 1:	2A, B

- Objective 5: Subtract a one-digit whole number from a whole number less than 20.

Prekindergarten:	Cognition Development 2D, 3B, 4C
Kindergarten:	2
Grade 1:	2A, B

Skill Area: Problem Solving

- Objective 6: Solve a one-step word problem involving addition/subtraction of whole numbers.

Kindergarten:	3A, B
Grade 1:	3A, B, C, D

---

*Administrators, counselors, supervisors, and instructors are reminded that prekindergarten and kindergarten attendance is optional. Students entering Grade 1 may not have had any exposure to the prerequisite concepts and skills they need to succeed prior to Grade 1.

Skill Area: Measurement

- Objective 7: Identify the correct time to the hour or half-hour.

Kindergarten: 4C
Grade 1: 4Bii

Skill Area: Geometry

- Objective 8: Identify a named geometric shape.

Kindergarten: 5A, B
Grade 1: 5

Grade 3

Skill Area: Number Concepts

- Objective 1: Arrange a group of three whole numbers from least to greatest or greatest to least.

Grade 1: 1Aiii, 6A
Grade 2: 1Ai
Grade 3: 1A

- Objective 2: Identify the place value for a given digit of a three- or four-digit whole number.

Grade 1: 1B
Grade 2: 1B
Grade 3: 1C

- Objective 3: Complete a pattern involving multiples of 2, 3, 4, 5, or 10.

Grade 1: 1Aii
Grade 2: 1Aii
Grade 3: 1B

- Objective 4: Express whole numbers in expanded notation.

Grade 3: 1C

- Objective 5: Use pictorial models to identify fractional parts of a whole or of a group of like objects.

Grade 1: 1C
Grade 2: 1C
Grade 3: 1E

Skill Area: Computation

- Objective 6: Add whole numbers having as many as three digits, with or without regrouping.

Grade 1: 2B
Grade 2: 2A
Grade 3: 2A

- Objective 7: Subtract whole numbers having as many as three digits, with or without regrouping.

Grade 1: 2B
Grade 2: 2A
Grade 3: 2A

Skill Area: Problem Solving

- Objective 8: Solve one-step word problems involving addition of whole numbers.

Grade 1: 2A, 3A, 3B, 3D
Grade 2: 3A
Grade 3: 2A, 3A, 3B

- Objective 9: Solve one-step word problems involving subtraction of whole numbers.

Grade 1: 2A, 3A, 3B, 3C, 3D
Grade 2: 3A
Grade 3: 2A, 3A, 3B

Skill Area: Measurement

- Objective 10: Select the unit of measure used to determine length, weight/mass, or capacity/volume.

Grade 1: 4A, B
Grade 2: 4A, B
Grade 3: 4A, B

- Objective 11: Identify pictorial models of two- and three-dimensional shapes.

Grade 1: 5
Grade 2: 5
Grade 3: 5

Grade 5

Skill Area: Number Concepts

- Objective 1: Interpret place value in whole numbers and decimals having no more than five digits.

Grade 3: 1C, D
Grade 4: 1C, D
Grade 5: 1D, E

- Objective 2: Identify equivalent fractions.

Grade 3: 1F
Grade 4: 1E, F
Grade 5: 1F, G

Skill Area: Computation

- Objective 3: Add and subtract decimals that have no more than five digits.

Grade 3: 2A, B
Grade 4: 2A
Grade 5: 2A, B, C

- Objective 4: Multiply whole numbers less than 100.

Grade 3: 2C, 6B
Grade 4: 2B
Grade 5: 2A, C

- Objective 5: Divide whole numbers that have four or fewer digits.

Grade 3: 2D
Grade 4: 2C
Grade 5: 2A, C

Skill Area: Problem Solving

- Objective 6: Solve two-step word problems involving addition/subtraction of whole numbers.
  - Grade 3: 3A, B
  - Grade 4: 3A, B
  - Grade 5: 3A, B

- Objective 7: Solve one-step word problems involving multiplication/division of whole numbers.
  - Grade 3: 3A, B
  - Grade 4: 3A, B
  - Grade 5: 3A, B

- Objective 8: Solve one-step word problems involving addition/subtraction of decimals.
  - Grade 3: 3A, B
  - Grade 4: 3A, B
  - Grade 5: 3A, B

Skill Area: Measurement

- Objective 9: Estimate measurements in metric and customary units.
  - Grade 3: 4A, B
  - Grade 4: 4A, B, C
  - Grade 5: 4A, B, C

- Objective 10: Interpret graphs.
  - Grade 3: 3D/7A, B
  - Grade 4: 6/7A, B, C, D
  - Grade 5: 6/7A, B, C, D

Skill Area: Geometry

- Objective 11: Find the perimeter or area of polygons using grids.
  - Grade 3: 4C
  - Grade 4: 4D, E
  - Grade 5: 4D, E

Grade 7

Skill Area: Number Concepts

- Objective 1: Convert decimals or percents to equivalent forms.
  - Grade 5: 1E
  - Grade 6: 1E
  - Grade 7: 1G

Skill Area: Computation

- Objective 2: Add and subtract fractions with like or unlike denominators.
  - Grade 5: 1F, 1G, 2B, 2C
  - Grade 6: 2B, 2C
  - Grade 7: 2A

- Objective 3: Add, subtract, and multiply decimals.

Grade 5: 1E, 2B, 2C
Grade 6: 2B, 2C
Grade 7: 1G, 3G

Skill Area: Problem solving

- Objective 4: Solve two-step word problems using the basic operations with whole numbers.

Grade 5: 3B, 3D
Grade 6: 3B
Grade 7: 3C, 3F, 3G, 7E

- Objective 5: Solve two-step word problems using addition, subtraction, and multiplication with decimals.

Grade 5: 3B
Grade 6: 3B, 4B
Grade 7: 3C, 3E, 3F, 3G

Skill Area: Measurement

- Objective 6: Convert measurements within either the metric or customary system of measurement.

Grade 5: 4C
Grade 6: 4B
Grade 7: 4A, 4B

Skill Area: Geometry

- Objective 7: Identify common geometric terms and figures.

Grade 5: 3C, 4D, 4E, 4F, 5A, 5B
Grade 6: 4C, 4D, 5A
Grade 7: 4C, 4D, 5A, 5B, 5C

- Objective 8: Find the perimeter of polygons.

Grade 5: 3C, 4B, 4D
Grade 6: 3D, 4B, 4C
Grade 7: 3D, 4A, 4C

Skill Area: Probability, Statistics, and Graphing

- Objective 9: Use information from charts or graphs to solve word problems.

Grade 5: 3D
Grade 6: 3E
Grade 7: 6B, 7E

- Objective 10: Determine the number of possible outcomes for a given situation.

Grade 5: 7C, 7D
Grade 6: 7C, 7D
Grade 7: 7D, 7E

- Objective 11: Solve one-step equations.

Grade 6: 3C
Grade 7: 3B

Grade 9

Skill Area: Number Concepts

- Objective 1: Identify the relationship between decimals, fractions, and percents.

  Grade 7: 1G, 3F
  Grade 8: 1F
  FOM: 1C, 3B, 3D

Skill Area: Computation

- Objective 2: Add and subtract fractions.

  Grade 7: 1G, 2A
  Grade 8: 2B
  FOM: 3B, 3C

- Objective 3: Multiply and divide decimals.

  Grade 7: 1G, 3F, 3G
  Grade 8: 3H, 8A
  FOM: 3E

Skill Area: Problem Solving

- Objective 4: Solve two- and three-step word problems using the basic operations.

  Grade 7: 3C, 3E
  Grade 8: 3B, 3C, 3D, 3E
  FOM: 1B, 3A, 3B, 3C, 3E

- Objective 5: Solve word problems using ratios, proportions, and percents.

  Grade 7: 3F
  Grade 8: 1F, 3E
  FOM: 1C, 4A, 4B, 4C, 4D, 5H

- Objective 6: Solve personal finance problems.

  Grade 7: 3G
  Grade 8: 3H
  FOM: 4A, 4B, 4C, 4D, 5H, 6F

Skill Area: Measurement

- Objective 7: Solve word problems involving metric/customary measurements using the basic operations.

  Grade 7: 3D, 4A, 4E
  Grade 8: 3C, 3D, 4A, 4D, 4E
  FOM: 1B, 2A, 5C, 5D

Skill Area: Geometry

- Objective 8: Find the areas of rectangles and triangles.

  Grade 7: 3D, 4D
  Grade 8: 3D, 4D
  FOM: 1B, 2A, 5D

Skill Area: Probability, Statistics, and Graphing

- Objective 9: Determine simple probabilities.

Grade 7: 7D
Grade 8: 7A, 7C, 7G
FOM: 6A, 6D
Cons. Math: 3A, 3D

- Objective 10: Use information from charts or graphs to solve word problems.

Grade 7: 6B, 7E
Grade 8: 7D, 7G
FOM: 5F, 6F, CM-1A, 1D
Cons. Math: 1A

Skill Area: Algebraic Concepts

- Objective 11: Use formulas to solve problems.

Grade 7: 3B, 4C, 4D, 4E
Grade 8: 3B, 3C, 3D
FOM: 5F, 6F, CM-1A

Exit Level

Skill Area: Number Concepts

- Objective 1: Select the set of numbers ordered from least to greatest.

Grade 8: 1G
Pre-Algebra: 2A

- Objective 2: Round numbers to a particular place value.

Grade 8: 1B

- Objective 3: Identify equivalent fractions, decimals, and percents.

Grade 8: 2A
FOM: 3B

- Objective 4: Convert numbers from exponential notation to standard notation.

Grade 8: 1A
Pre-Algebra: 2C
Algebra I: 1E

Skill Area: Computation

- Objective 5: Solve problems involving addition/subtraction/multiplication of fractions and mixed numbers.

Grade 8: 2B
FOM: 3C
Pre-Algebra: 2A

- Objective 6: Use the basic operations to solve decimal problems.

Grade 8: 2B,
FOM: 3E
Pre-Algebra: 2A

- Objective 7: Solve problems involving addition of integers.

Grade 8:	1G
FOM:	3A
Pre-Algebra:	2A
Algebra I:	1D

Skill Area: Problem Solving

- Objective 8: Solve word problems involving multiple operations on whole numbers, decimals, fractions, and mixed numbers.

Grade 8:	3A, 3B, 3E, 3H
FOM:	3A
Consumer Math:	2A, 2B, 2C
Pre-Algebra:	3A, 3B, 3C, 3D

- Objective 9: Solve word problems involving proportions.

Grade 8:	3E
FOM:	5E
Consumer Math:	2A, 2B, 2C

- Objective 10: Solve word problems involving percent.

Grade 8:	3H
FOM:	5E
Consumer Math:	2A, 2B, 2C
Pre-Algebra:	3C

Skill Area: Measurement

- Objective 11: Solve problems involving metric/customary measurements using the basic operations.

Grade 8:	4A, 4B, 4C, 4D, 4E
FOM:	1A, 1B, 1C
Pre-Algebra:	4D

Skill Area: Geometry

- Objective 12: Solve problems involving geometric formulas.

Grade 8:	4D, 4E, 5A, 5B
FOM:	2A
Informal Geometry:	10B
Geometry:	2A

- Objective 13: Use geometric properties to solve problems involving geometric shapes.

Grade 8:	5A, 5B, 5C
FOM:	2A,
Pre-Algebra:	4D
Informal Geometry:	2C
Algebra I:	2A

Skill Area: Probability, Statistics, and Graphing

- Objective 14: Solve word problems involving averages.

  Grade 8: 7E
  FOM: 6C
  Consumer Math: 1D
  Pre-Algebra: 5D

- Objective 15: Solve word problems involving simple probability.

  Grade 8: 7C, 7F, 7G
  FOM: 5G, 6E
  Consumer Math: 3A, 3B, 3C, 3D
  Pre-Algebra: 5A, 5B, 5C

- Objective 16: Use information from graphs and tables to solve word problems.

  Grade 8: 7D
  FOM: 5F
  Consumer Math: 1A
  Pre-Algebra: 5F

Skill Area: Algebraic Concepts

- Objective 17: Solve word problems using formulas.

  Grade 8: 3B, 4D
  FOM: 5B
  Pre-Algebra: 3A
  Algebra I: 2F
  Informal Geometry: 10B, 7C, 10D

- Objective 18: Solve problems to determine the value of a variable.

  Grade 8: 3B
  FOM: 5B
  Pre-Algebra: 3A
  Algebra I: 2F

Evaluation

Mathematics concepts and skills should be presented to students sequentially, deliberately connecting concrete experiences to pictorial and finally abstract representations. Instruction in mathematics necessarily begins with a heavy emphasis on manipulative activities. While this is especially true in the primary grades where many concepts are introduced, teaching with manipulatives is also appropriate throughout the elementary and secondary grades wherever new concepts are presented. The length of time necessary for students to spend on concrete activities may vary, depending on the student and on the concept. Generally, however, teachers tend to spend too little time here, skipping instead to "how to do it" before students fully understand the underlying concept.

Evaluation methods must be consistent with instructional approaches and matched to students' developmental levels. At the concrete introductory levels, evaluation of mastery should consist of student demonstration using manipulative materials. At other points in the instructional sequence, evaluation may take many forms, from observation to abstract pencil-and-paper tests, depending on the concept being tested.

Teachers may assess students by working with them in small groups, on a one-to-one basis, or by examining a project. Assistance can be provided by parent volunteers, older students, teacher interns, and other staff.

Introductory computation concepts, in particular, should be evaluated by student demonstration with manipulative materials. Although many districts require students to pass timed tests on basic computation facts, speed tests should not be used to evaluate mastery even at the abstract level. Such tests likely increase students' mathematics anxiety and reinforce the view that mathematics consists only of quick fact recall. Where speed is considered important, teachers should use techniques that do not force students to compete with each other but allow them to work toward improving individual time for completion.

The essential elements reflect a wide range of content beyond computation, with the major focus of the mathematics program on solving problems using mathematics. Students should be able to use a variety of problem-solving tools in learning and in demonstrating mastery. Tools such as calculators, computers, measurement devices, and manipulative models should be used regularly in instruction as well as in evaluation of problem-solving skills.

In accordance with State Board of Education rules, schools must maintain data on student mastery. Methods of record keeping may range from simple checklists to comprehensive records detailing diagnosed weaknesses. Once an assessment has been made and the records are complete, decisions must be made concerning enrichment and/or remediation activities.

On the scope and sequence chart on pages 71-73, subelements are starred(★) to provide assistance in evaluating mastery of the essential elements and subelements. Subelements are coded as follows:

★ Initial development of concepts using models and manipulatives should be evaluated concretely in much the same way the concepts are taught. These concepts should generally be evaluated for mastery using models and manipulatives rather than paper-and-pencil tests.

★★ In-depth instruction connects the concrete understanding of a concept to the abstract. Instruction at this level is comprehensive. These subelements may be evaluated for mastery by students' using written materials or through oral demonstration of understanding.

★★★ Concepts are extended, e.g., addition of whole numbers to addition of decimals. Since some review is built into this level, caution against overteaching should be exercised. These subelements may be evaluated in a variety of ways, depending on the concept.

Scope and Sequence Chart

NOTE: Insert chart as page 71 inside this publication.

Mathematics

Scope and Sequence Chart

Essential Elements	K	1	2	3	4	5
Concepts and skills associated with the understanding of numbers (whole, interger, and non-negative rational) and the place-value system	Experience at the readiness level: • number concepts★ • one-to-one correspondence★ • ordering★ • counting (objects)★	Acquire number concepts through: • counting (objects)★★ • skip counting★ • comparing and ordering★ Use tens and ones to show place value★ Recognize fractions (models)	Acquire number concepts through: • comparing and ordering★★ • skip counting★★ • investigating odds and evens★★ Use hundreds, tens, and ones to show place value★★ ★	Compare and order★★★ Use odds, evens, and skip counting★★★ Use place value★★★ Identify decimal place value using money (tenths and hundredths)★ Use fractions (models)★ Recognize equivalent fractions (using models)★ Read and write whole numbers★★	★★★ Round numbers★ ★★★ Use decimals (place value)★★ ★★ ★★ Read and write whole numbers and decimals★★★	★★★ Use exponential notation★ ★★ ★★★ Use decimals★★★ ★★★ Identify equivalent fractions★★★
The basic operations on numbers (addition, subtraction, multiplication, division), their properties, and their uses	Experience at the readiness level: • physical representations of addition and subtraction★	Acquire number concepts of joining and separating using manipulatives★ Add and subtract whole numbers★	★★ Demonstrate multiplication concepts of whole numbers (using concrete materials)★	Add and subtract decimals using money★ ★★★ Multiply whole numbers★★ Divide whole numbers (using concrete materials)★ Recognize inverse operations★	Add and subtract whole numbers and decimals★ ★★★ Divide whole numbers★★ Use inverse operations★ Add and subtract fractions using models (with like denominators)★	Add, subtract, multiply, and divide whole numbers★★★ Add, subtract, and multiply decimals and fractions (not including mixed numbers)★ ★★
Experience in solving problems by selecting and matching strategies to given situations	Experience at the readiness level: • oral problem solving with objects and stories★ • identifying patterns★	Apply addition and subtraction★ Identify story situations depicting the joining and separating process★ Draw pictures depicting the joining and separating process★ Use modeling for presenting and solving problems★	★★ Use charts and graphs★	Estimate with whole numbers★ Use basic operations with whole numbers and with decimals using money★ Use geometry★ ★★	Estimate with whole numbers and decimals★ Use basic operations with whole numbers, decimals, and fractions★ ★★ ★★	★★ ★★ Apply geometric concepts★ ★★★
Investigating the use of computing devices and their capabilities						

NOTE: See page 70 for explanation of stars.

6	7	8
Study the history of numeration systems★ → ★★	Investigate the history of numeration systems★★ ★★	★★★ →
Round whole numbers and decimals★★	★★★	★★★ →
Factor (primes and composites)★	★★	★★★ →
★★★ →	Use scientific notation★	★★ →
Find relationships between fractions, decimals, percents, and ratio★	Use fractions, decimals, ratio, and percent★★	Use ratio and percent★★★ →
Find the least common multiple★	★★ →	
Find the greatest common factor★	★★ →	Determine significant digits★★
Use integers★	★★	★★ →
Add, subtract, multiply, and divide integers★	Add, subtract, multiply, and divide rational numbers★★	★★★ →
	Use exponents★	Demonstrate rational number properties★
		Find the absolute value★
★★★ →		
Add, subtract, multiply, and divide fractions and decimals★★★		
★★ →	Estimate solutions★★	★★ →
★★	★★★ →	
Solve simple linear equations★	Solve simple linear equations and inequalities★	Write and solve equations★ →
★★	★★	★★★ →
★★★ →		Use graphs★★★
	Use denominate numbers in problem solving★	★★ →
	Use proportion and percent★★	★★ →
	Practice skills in personal finance★	★★ →
		Use probability and statistics★★
	Use calculators and/or computers to solve mathematics problems★	★★ →
		Determine limitations and strengths★

Scope and Sequence Chart

NOTE: Insert chart as page 73 inside this publication.

Mathematics *(continued)*

Essential Elements	K	1
Measurement concepts and skills using metric and customary units	Experience at the readiness level: • comparing ★ • identifying money ★ • identifying time units ★	Measure (using measuring instruments) ★ Use measurement units for: • money ★ • time ★ • length ★ • weight/mass ★ • liquid volume ★ • temperature ★
Properties and relationships of geometric shapes and their applications	Experience at the readiness level: • classification ★ • size and shape ★ • patterns ★	Recognize and name models of two-dimensional shapes ★
The representation of numbers on a line and pairs of numbers on a coordinate plane	Experience at the readiness level: • ordering points on a line ★	Order numbers ★ Locate points on a number line ★
The use of probability and statistics to collect and interpret data	Experience at the readiness level using objects: • classification ★ • ordering ★	Classify objects ★ Order numbers ★

NOTE: See page 70 for explanation of stars.

2	3	4	5	6	7	8
★	★	★	★	★ →		
★★ →	Solve problems using metric and customary unit measures (no conversion)★	★★	Use metric and customary units in applications (no conversion between systems of measure)★★	★★★	★★★	★★★ →
		Find relationships between metric units★	Convert units within the metric system★★ →		Convert within the metric system★★★	★★★ →
	Find perimeter★	★★ →	Find perimeter and circumference★★/★	★★★	★★★ →	Recognize precision★★
		Find area (with grids)★	★★ →	Find area, surface area, and volume★★	Apply skills★★	★★ →
			Find volume (with cubes)★		★★	★★★ →
Recognize and name models of two- and three-dimensional shapes★	Identify characteristics of two- and three-dimensional shapes★★	Investigate the properties of two- and three-dimensional figures★★	Use the properties of two- and three-dimensional figures★★★	★★★ →	Understand properties and definition of two- and three-dimensional figures★★	
					Use points to define lines, rays, angles, and planes★	
		Work with similarity, congruence, and symmetry★	★★ →	Apply similarity, congruence, and symmetry in problem situations★★★	★★★ →	
			Find surface area (with models)★		Construct geometric figures★	★★ →
				Study translations, reflections, and rotations and their applications★		Use right triangle geometry with: • Pythagorean property★ • similarity★ • indirect measurement★ • ratios★
					Apply skills★★	★★ →
★★	★★ →	Draw number lines with fractions (with like denominators) and decimals★★	Draw number lines with fractions and decimals★★	Graph points on a coordinate plane★	Graph on a coordinate plane: • similar figures★★ • reflection images★★ • linear equations★★	Graph on a coordinate plane: • translations and reflections★★ • nonlinear graphs★★ ★★ →
Work with addition tables★★						
					Apply skills★★	★★ →
	Work with addition and multiplication tables★★					
Collect and organize information★	★ →	Collect data and use to construct graphs★	★★	★★ →		
Read bar graphs★	Interpret and construct picture and bar graphs★★	Interpret pictographs and use information to solve story problems★	Interpret and construct charts and graphs★	★★	★★★ →	Use tables and graphs★★★
Make picture graphs★		List the possible outcomes in a given situation★	Use counting arrangments★	Construct sample spaces★★	Find the mean, median and mode★	★★ →
		Read and interpret line graphs★	Predict possible outcomes from a sample★	Determine and interpret fractional probability★	Use simple event probability★★	Draw Venn diagrams★
				Find averages★	Use frequency distributions★	Make predictions★★
				Apply probability and statistics concepts to solve problems★★		Find the probability of simple and compound events★★
						Use counting methods and factorials★
					Apply skills★★	★★ →

1986

Reteaching and Remediation

Students are expected to master what is taught. When they don't, educators have the responsibility to provide reteaching so that student deficiencies are immediately remediated. In this way, students can succeed in future learning. Reteaching may be thought of as remediation provided shortly after direct instruction. Effective reteaching can prevent the need for in-depth remediation later. Individualized reteaching for every student would be ideal but may not be practical.

Teachers should plan for reteaching at the same time they are planning initial instruction and enrichment. Reteaching activities should be different from those used for initial instruction and might even involve an explanation from another teacher, a student, or a trained parent volunteer. Reteaching may be structured in many ways. A reteach day can be planned every week or two, or several days might be set aside every few weeks for reteaching. Activities can be set up in a learning center or may be provided through structured group activities. Teachers and administrators will want to examine several possibilities in making decisions on reteaching.

One powerful tool that can help teachers plan reteaching and remediation is assessment data. Such data reveal specific areas of student need where reteaching is necessary.

Elementary Grades

Remediation is an integral part of the elementary mathematics program. Districts may have policies that allow students to move on to the next grade even though they are functioning below grade level in mathematics. By making such a decision, administrators commit themselves to securing special attention for these students in mathematics. They may be provided extra mathematics instruction, may be placed in a smaller mathematics class, may be assigned help from support staff, or may receive other assistance as defined by local policy.

Instruction for students who are functioning below grade level should focus on areas not mastered from the previous grade. These concepts and skills should be combined, where possible, with on-grade level concepts and skills. Reteaching is expected for any student who does not demonstrate mastery of a particular concept or skill.

Grades 7-8

Remediation and reteaching are especially important at the middle school/junior high level if students are to be able to complete a full four years of academic mathematics in high school. This type of preparation is essential for a student to have options and choices of careers or college majors. As in the elementary program, if local policy allows students to progress to the next grade while functioning below grade level in mathematics, the district should provide extra assistance to those students through smaller class size, additional time spent in direct mathematics instruction, help from support staff, a combination of these possibilities, or other assistance as defined by local policy. Reteaching should occur in any area where a student has not demonstrated mastery. It may be structured to occur during regular mathematics time or outside of class time and should involve a different approach from that used during initial instruction.

Grades 9-12

Students who have not yet mastered seventh and eighth grade skills upon entering Grade 9 have several options available. Two units of fundamental, consumer-type mathematics provide a review of basic skills on a wide variety of topics and present applications of these skills in consumer situations. The two courses, Fundamentals of Mathematics and Consumer Mathematics, stress remediation of specific deficiencies so that students may demonstrate mastery on the Texas Educational Assessment of Minimal Skills tests and progress into Pre-Algebra at the conclusion of either course. These classes should be kept small, and teachers should use innovative materials and teaching strategies to enable students to master skills they lack.

Students functioning below grade level have rarely been directed into Pre-Algebra or Algebra I in the past. This program is intended to encourage as many students as possible to take Algebra I. One year of high school algebra is becoming increasingly important both in salaries and in future career options for graduating high school students. Rather than providing a third year of the same content of a basic skills course, directing students into Pre-Algebra and Algebra I should more nearly fulfill their educational needs.

As an increased effort toward mastery of skills at each level is made, the number of students entering high school in the below-grade-level courses should continue to decrease. Students should be encouraged to remediate their deficiencies early and to progress on toward more academically rigorous courses.

Enrichment

Not only is remediation and reteaching an important part of instruction, but equally important are enrichment activities from which all students can benefit. These activities should be designed to extend skills in logical thinking, strategic reasoning, estimation, mental arithmetic, and problem solving. Enrichment activities should not be just more or harder problems but problems that expand upon the ideas presented in the traditional curriculum.

Enrichment activities can be provided through the use of learning centers, grouping within the classroom, whole class projects, and individualized programs. Other examples appropriate for enrichment activities include independent research projects, games that require the use of strategies or higher level thinking skills, and logic problems that require students to recognize patterns and apply sequencing skills.

Assignment of Teachers

Elementary

Under current certification standards, a teacher may be certified to teach elementary school with little or no mathematics training. To make the best use of those staff members who do have significant training in mathematics, the staff may want to departmentalize to ensure that appropriately trained teachers are teaching mathematics. Staff members may also find it advisable to chose a team leader or other teachers committed to teaching mathematics for retraining to add an area of specialization in mathematics to their certifications.

Secondary

Effective secondary programs in mathematics reflect an ongoing effort to upgrade the training of the mathematics teachers. Where districts find it necessary to assign several teachers on Temporary Classroom Assignment Permits or Emergency Permits, instructors with the greatest number of courses in mathematics who have a commitment to mathematics education should be supported and retrained to gain a teaching field in mathematics.

Special Concerns Related to Calculus and Computer Mathematics

A secondary-certified mathematics teacher may teach any high school mathematics course. For calculus and computer mathematics, however, certain specialized coursework is desirable.

When teachers are assigned to teach calculus, they should have a strong mathematics background including calculus, multivariable calculus, and differential equations. Also, workshops are available that provide information for the teaching of advanced placement calculus.

A computer mathematics teacher ideally should have at least nine semester hours in computer-related coursework. This need not be computer science but should include some programming and appropriate additional coursework.

Staff Development In Mathematics

Identification of Needs

A needs assessment should be made to determine the most critical areas for staff development. The assessment may be formal or informal and should incorporate existing recent information wherever possible. Input may be provided by examining certification records, obtaining written input of teachers, and involving small groups of teachers in informal verbal discussion. If gathering local data is not practical, statewide or regional data may be used for general direction. Areas to be addressed include the number of teachers who are teaching mathematics without adequate preparation, the college mathematics preparation of uncertified and certified teachers, areas teachers feel least prepared to teach, areas not well addressed by existing curriculum documents or resource materials, and areas where the use of manipulative materials is not well understood.

Planning

A major goal of local staff development efforts should be to give teachers a solid foundation of content relevant to the mathematics essential elements. Teachers should be actively involved in the formulating of long- and short-range plans for their staff development.

The plans may be developed locally, or educators may use modules that are being developed by the Texas Education Agency. A list of certified trainers will be available from the Agency and from education service centers as the modules are completed and ready for implementation.

To help teachers implement the essential elements, staff development should provide a solid understanding of content at the appropriate grade level and beyond. It should also provide a wealth of effective teaching strategies for the discipline. Concept development and the effective integration of manipulative activities into classroom instruction should be emphasized.

In addition to providing a foundation for teaching the essential elements, there is a need for numerous other staff development efforts. Districts are encouraged to provide staff development related to management of the mathematics class, innovative teaching techniques, broad problem-solving skills across content areas, and specially developed programs. Examples of successful creative efforts in content and in delivery systems developed by school districts will be disseminated by the Texas Education Agency to allow other districts or regions to replicate those successes.

Trainers

Trainers should be individuals who are well-trained in mathematics, who have classroom teaching experience, who are thoroughly familiar with the content of the essential elements, and who are, thereby, well qualified to present the material. In some cases, pairs of trainers representing different backgrounds such as a teacher, supervisor, consultant, and university educator may be preferred.

Structure

For staff development to count toward Advanced Academic Training, AAT guidelines should be followed carefully. Workshops may be scheduled during evenings, weekends, or summers and must have prior approval by the Texas Education Agency and the school district. In addition to workshops, seminars, and courses, professional mathematics conferences (national, state, and local) may provide an excellent vehicle for teacher staff development.

Follow-Up

Follow-up activities with teachers are critically important in implementing change. Follow-up procedures should be structured during the initial planning of the staff development sessions and may include classroom visits, small group debriefing sessions, follow-up workshops to extend what has been learned, and peer observation.

The Challenge

In summary, the expectation of the Chapter 75 mathematics curriculum is that instruction will be focused on teaching for understanding. Students will be actively involved in learning through experiences with manipulative materials. Drill and practice activities will be used only after students have developed the underlying concepts. Students will apply skills and concepts in problem solving situations, both routine and nonroutine, to extend skills and to develop higher-level thinking skills. All students will be encouraged to choose a complete mathematics program in high school to allow them options after graduation.

A statewide effort is underway to improve the teaching of mathematics at all grade levels. This effort requires a philosophical commitment, as well as a commitment of time and money. Even where excellent instruction already exists, teachers are being asked to upgrade that instruction. The program emphasizes mastery of concepts and skills, reteaching for students unable to demonstrate mastery, and enrichment for all students whenever possible. School districts are to maintain academic expectations for students at all grade levels. The end result of this effort should be that all students who graduate from Texas schools will have an improved mathematics education.

What lies ahead is a challenge to all involved in mathematics education in Texas to continue the commitment to improve significantly the teaching and learning of mathematics across the state.

Elementary and Middle School Mathematics

Philosophy of the Elementary Mathematics Program

Descriptions of the elementary mathematics program in Chapter 75 and in textbook proclamations emphasize a philosophy of instruction based on current research. Neither the New Math's heavy emphasis on vocabulary and abstract concepts nor the narrow interpretation of mathematics as arithmetic in the Back-to-Basics movement has brought mathematics education to the high standards society demands. The Chapter 75 mathematics program includes critical components of both of these efforts within a total framework that stresses understanding, experience, and application of concepts and skills.

Students develop mathematical concepts through concrete experiences with manipulative materials. The development of concepts and skills from the concrete level to the connecting level and then to the abstract level is accomplished through activities deliberately designed to help students make the transition. Applying skills in problem-solving situations is a priority in all mathematics instruction. Experience in estimating and recognizing answers as reasonable or unreasonable is included.

The content of the curriculum includes concepts and skills from the range of mathematics topics, including measurement, geometry, graphing, probability and statistics, numeration, our number system and its properties, and computation. Problem solving is not only included as a separate essential element but is also interwoven as an integral part throughout all the essential elements in the mathematics curriculum. The curriculum requires that all students will have the opportunity to experience and master all of the identified areas of mathematics, as described in the essential elements for a well-balanced program.

Philosophy of the Mathematics Program, Grades 7-8

Courses listed for mathematics, Grades 7-8, include a review and extension of skills from the elementary grades and develop additional concepts and skills in topics previously mastered. The use of manipulative materials to develop concepts continues to be encouraged. Experience with calculators and computers is to be integrated as part of both Grade 7 and Grade 8 programs, above and beyond the separate semester of Computer Literacy required at this level.

A number of topics are presented at the seventh and eighth grade to prepare students for high school mathematics, especially for algebra and geometry. Students should be provided with experiences specifically designed to help them make the transition from the concrete and symbolic levels to an abstract level of reasoning that will be necessary for future success in mathematics. Most students who master the content of seventh and eighth grade mathematics will be prepared to succeed in Algebra I in Grade 9.

The content of the curriculum includes concepts and skills from the range of mathematics topics such as measurement, geometry, graphing, probability and statistics, numeration, our number system and its properties, and computation. Problem solving is not only included as a separate essential element but is also interwoven as an integral part throughout all the essential elements. The curriculum requires that all students will have the opportunity to experience and master all the identified areas of mathematics described in the essential elements.

Recommended Minimum Manipulative Materials

The materials listed below are recommended minimum sets that should be available in each classroom for the grade level indicated. Descriptions of items are provided after the list. Materials may already be in many classrooms, and teachers may be able to construct a few items from readily available materials. Listed items are certainly not the only materials that are appropriate. The list is intended to provide guidelines, especially for districts that have not conducted an in-depth needs assessment for manipulative materials.

The mathematics curriculum is focused on concept and skill development through students' experience with manipulatives, which are essential classroom instructional materials. Ideally, a district should develop a plan that would provide for the orderly purchase of manipulative materials as well as for appropriate staff development on how to use the materials.

Where quantities are listed, they represent the recommended minimum number of items needed for a class size of 22-25 students. Translucent plastic models of many manipulatives are available for teachers to use for demonstration on an overhead projector. The materials that have translucent versions available are noted with an asterisk (*).

List of Materials

Kindergarten

- Interlocking counting cubes (1000)
- Attribute blocks (four sets—60 each)
- Pattern blocks (four sets—250 each with two mirrors)*
- Buttons, shells, keys, and other familiar objects for counting and sorting
- Measuring instruments for time (demonstration clock)
- Play money

Grade 1

- Interlocking counting cubes (1000)
- Attribute blocks (four sets—60 each)
- Pattern blocks (four sets—250 pieces each with two mirrors)
- Measuring instruments (to measure length, volume, mass, temperature, and time)*
- Play money*

Grade 2

- Interlocking counting cubes (1000)
- Base ten blocks (ones, tens, and hundreds for each child)*
- Attribute blocks (four sets—60 each)
- Pattern blocks* (four sets—250 pieces each with two mirrors)

- Measuring instruments, both metric and customary (to measure length, volume, mass, temperature, and time)*
- Play money*
- Two- and three-dimensional geometric models (two sets of common shapes and solids)*

Grade 3

- Interlocking counting cubes (1000)
- Base ten blocks (ones, tens, and hundreds for each child, one thousand-block for each four children)*
- Pattern blocks (four sets—250 pieces each with two mirrors)*
- Fraction models—circles, squares, bars, and/or rods*
- Measuring instruments, both metric and customary (to measure length, volume, mass, temperature, and time)*
- Play money
- Two- and three-dimensional geometric models (two sets of common shapes and solids)*

Grade 4

- Base ten blocks or other place value models (ones, tens, and hundreds for each child; one thousand-block for each four children)*
- Decimal models*
- Fraction models—circles, squares, bars, and/or rods*
- Measuring instruments, both metric and customary (to measure length, volume, mass, temperature)*
- Interlocking centimeter cubes (2000)
- Tangrams—one per child*
- Two- and three-dimensional geometric models (two sets of common solids and shapes)*
- Geoboards—one per child and one for teacher demonstration*

Grade 5

- Base ten blocks or other place value models (ones, tens, and hundreds for each child; one thousand-block for each four children)*
- Decimal models*
- Fraction models—circles, squares, bars, and/or rods*
- Measuring instruments, both metric and customary (to measure length, volume, mass, temperature)*
- Interlocking centimeter cubes—2000
- Tangrams—one per child*
- Two- and three-dimensional geometric models (two sets of common solids and shapes)*
- Geoboards—one per child and one for teacher demonstration*

Grades 6-8

- Decimal models*
- Fraction models—circles, squares, bars and/or rods*

- Measuring instruments, both metric and customary (to measure length, volume, mass, temperature)*
- Tangrams—one per student*
- Geoboards—one per student and one for teacher demonstration*
- Geometric models*
- Probability devices—spinners, regular solids
- Calculators*

Description of Listed Materials

Attribute Blocks

Wooden or plastic geometric shapes in various colors, shapes, and sizes. These are used for sorting, classifying, and comparing, as well as for problem solving.

Base Ten Blocks

Wooden or plastic models of ones, tens, hundreds, and thousands. These are to be used in numeration experiences and to demonstrate place value concepts. Other place value models, including chips for trading, may also be used.

Buttons, shells, keys, or objects for counting and sorting

Simple, safe objects for very young learners to use in developing numeration skills. Objects should have both similarities and differences.

Calculators

Handheld calculators for each student with at least four functions. In addition, square root and memory functions are helpful. While calculators are not true manipulative materials, they are problem-solving tools. Plan for the purchase of calculators at the same time other mathematics materials are purchased.

Decimal Models

Wooden, cardboard, or plastic squares and rectangles divided into tenths, hundredths, and thousandths.

Fraction Models

Plastic, wood, or rubber models showing the same size whole divided into various fractional parts. Colored rods in graduated lengths may also be used.

Geoboards

Square plastic or wooden boards with a square array of pegs or nails, usually at least 5x5. Geoboards with a circular array of pegs can be used for circle properties at Grades 6-8. A clear plastic demonstration geoboard for the teacher is recommended. These are used to develop geometry and measurement concepts.

Geometric Models

Wooden or plastic models of basic shapes. For two-dimensional shapes, include squares, circles, triangles, rectangles, parallelograms and regular 5-, 6-, 8-, 10-, and 12-sided polygons; for three-dimensional models, include spheres, cubes, cones, rectangular solids, pyramids, various prisms, and regular solids. Large models suitable for measuring will be needed in Grades 6-8.

100

Interlocking Centimeter Cubes

Small plastic cubes, one centimeter on a side, which snap together. These generally have a mass of one gram and are used for mass, length, or volume activities. They may also be used for remediating numeration skills.

Interlocking Counting Cubes

Plastic cubes, approximately 2-cm or 1-inch square, which snap together to be used to demonstrate numeration concepts and basic computation concepts.

Measuring Instruments (including metric and customary units)

- Mass. For kindergarten, a simple balance for comparing objects as heavier or lighter. For Grades 1-8, balances or scales with units appropriate for the grade level.
- Length. Rulers, meter sticks, tape measures, and trundle wheels with graduated units appropriate for the grade level.
- Volume. Measuring cups, measuring spoons, and graduated containers of various shapes. Units should be appropriate for the grade level.
- Temperature. Thermometers of various types, including a large-size demonstration thermometer. Degree graduations should be appropriate for the grade level.
- Time. Clocks, one per student and a teacher demonstration clock all with movable hands.

Pattern Blocks

Brightly colored wooden or plastic blocks in various geometric shapes. These are used to develop concepts of patterning, fractions, and geometry.

Play Money

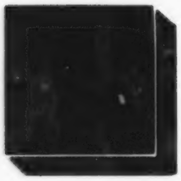
Realistic play money coins can be plastic. Paper money may be included in Grade 3. Real money may be used where appropriate.

Probability Devices

Spinners with various configurations; cubes or other regular geometric solids with colors, numbers, and letters on the faces. These are used to develop probability and statistics concepts.

Tangrams

A set of seven plastic, wooden, linoleum, or posterboard pieces constructed according to a specific pattern within a square. These are used to develop concepts in geometry, measurement, and problem solving.



High School Mathematics

Philosophy of the Mathematics Program, Grades 9-12

The philosophy of the high school mathematics program is to provide a sequence of courses that will prepare students for daily mathematics applications as well as for future vocational needs. Problem solving is an emphasis in all courses, and students successfully completing the program should have problem-solving skills which will be useful to them throughout life. The use of manipulative materials plays an important role in high school mathematics. Manipulatives aid in introducing new concepts and in remediating deficiencies.

The mathematics program is designed to provide appropriate course offerings for students at numerous levels of achievement and ability. Four courses are designed for students functioning below the expected level of achievement at the time they enter Grade 9. These courses are Fundamentals of Mathematics, Consumer Mathematics, Pre-Algebra, and Informal Geometry. Numerous other courses are available for students functioning at the expected level of achievement. These courses include Algebra I, Algebra II, Geometry, as well as more advanced courses such as Trigonometry, Elementary Analysis, Analytic Geometry, Precalculus, Calculus, Mathematics of Consumer Economics, Computer Mathematics I and II. Other advanced and independent study courses are available for students who require more specialized coursework.

High school graduation requirements have been increased to provide more and stronger mathematics preparation for all students. Although students can meet the three-unit graduation requirement in mathematics by taking all low-level courses, more students are anticipated to complete at least one unit of algebra prior to graduation. Students should be encouraged to take algebra, even if it means taking more than the three-unit requirement. Promotion, retention, and prerequisites are based on mastery of the essential elements for each course.

Mathematics Requirements for High School Graduation

High School Program

Of the 21 units required for high school graduation, three units of mathematics must be completed. The units may be selected from any state-approved mathematics course. If Algebra I has been satisfactorily completed in Grade 7 or 8, three additional units of mathematics must be completed.

Advanced High School Program

Of the 22 units required for graduation under the advanced high school program, students must complete Algebra I, Algebra II, and Geometry. If Algebra I has been satisfactorily completed prior to Grade 9, the student must complete Algebra II and Geometry and must select an additional unit from the following full-year or semester courses: Computer Mathematics I, Computer Mathematics II, Precalculus, Trigonometry, Elementary Analysis, Analytic Geometry, Probability and Statistics, Number Theory, History of Mathematics, Survey of Mathematics. Additional courses appropriate for students in the advanced high school program include: Calculus, Advanced Mathematics for Business, Linear Programming, Linear Algebra.

An additional requirement for the advanced high school program is one unit of computer science. This requirement may be satisfied through Computer Mathematics I or II, Computer Science I or II, Business Data Processing and Introduction to Computer Programming, Business or Vocational Word Processing, Vocational Computer Programming, or Vocational Data Processing. A single course cannot be used to satisfy two different requirements. Thus Computer Mathematics I can either be a mathematics unit or a computer science unit but not both.

Advanced High School Honors Program

Mathematics courses designated as honors courses may be developed by local districts in accordance with Title 19, Chapter 75.152 of the Texas Administrative Code. These courses, if approved by the Commissioner of Education, may be counted among the five required honors courses in the 22-unit advanced high school honors program. Calculus is designated as an honors course whenever offered.

Mathematics Courses, Grades 9-12

High school courses approved for mathematics credit are listed as follows. Courses marked "+" are required to be offered. Those designated as advanced are courses that students may take to meet requirements for the advanced high school program. Note that Algebra I, Algebra II, and Geometry are required for the advanced program. Advanced courses are available to any student with appropriate prerequisites.

+Fundamentals of Mathematics
+Consumer Mathematics
+Pre-Algebra
Informal Geometry
+Algebra I (advanced—required)
+Algebra II (advanced—required)
+Geometry (advanced—required)
Mathematics of Consumer Economics
Computer Mathematics I (advanced)
Computer Mathematics II (advanced)
*+Precalculus (advanced)
*Trigonometry (advanced)
*Elementary Analysis (advanced)
*Analytic Geometry (advanced)
Probability and Statistics (advanced)
Calculus (advanced)
Advanced Mathematics for Business (advanced)
History of Mathematics (advanced), independent study
Survey of Mathematics (advanced), independent study
Number Theory (advanced), independent study
Linear Algebra (advanced), independent study
Linear Programming (advanced), independent study

+Courses must be offered.

*Precalculus is listed as a course that must be offered unless Trigonometry and either Elementary Analysis or Analytic Geometry are offered in its place.

Planning Course Offerings to Meet Student Needs

Under State Board of Education rule, the following high school mathematics courses must be offered at least in alternating years:

Fundamentals of Mathematics

Consumer Mathematics

Pre-Algebra

Algebra I

Algebra II

Geometry

Precalculus (or Trigonometry and either Elementary Analysis or Analytic Geometry)

If a school district offers only the courses listed above, it will provide a course for every student. However, students will have no flexibility once they enter the sequence of courses. To provide options for students, especially for those experiencing minimal success, the following courses are strongly recommended at least in alternating years:

- Informal Geometry
- Mathematics of Consumer Economics
- Computer Mathematics I. Computer Mathematics is often considered a course for mathematically successful students. It is possible, however, to structure a more basic section of the course that relies almost totally on software, providing heavy emphasis on applications and reinforcing mathematics concepts.

Options for Small Schools

Chapter 75 allows districts to offer courses in alternate years. In mathematics, some courses lend themselves to alternate-year delivery; some courses would logically be offered every year to allow students to enroll in a complete sequence of courses.

Pairs of required courses that might be offered in alternate years are Fundamentals of Mathematics/Consumer Mathematics and Algebra II/Geometry. If Fundamentals of Mathematics and Consumer Mathematics are alternated, students will need to be carefully grouped in the classroom to accommodate their being at different levels. Since there is no state-required sequence between Algebra II and Geometry, these courses may be paired relatively easily. Teachers may need to make adjustments, however, for students with different prerequisite skills.

If a district offers the additional three courses recommended on page 00, Informal Geometry could be reasonably alternated with Mathematics of Consumer Economics. Also, Computer Mathematics I could be alternated with Precalculus, although this option may prevent some students from having access to Precalculus and would, therefore, not be desirable.

Other mathematics courses listed in Chapter 75 may be offered to meet special needs of students who are completing the requirements for the Advanced High School Program.

In addition to alternating courses, other options may be used to provide a complete range of courses for students. While these are not ideal solutions, they may be useful for small schools with one or two mathematics teachers.

One option is to teach two courses in the same classroom. For example, a few students enrolled in Informal Geometry may meet in the same room as a Geometry class. Each group needs to be taught the essential elements for the course they are taking. Some activities may overlap, however. Similarly, a small group of students in Mathematics of Consumer Economics may meet in the same classroom as Consumer Mathematics.

A second possibility is to provide students an opportunity to enroll in correspondence courses in a structured setting. As described above, a small group of students may work on Informal Geometry in a Geometry class. The students could be enrolled in a correspondence course for Informal Geometry, with only minimal assistance needed from the teacher. Likewise, students could be enrolled in a correspondence course on Mathematics of Consumer Economics with minimal teacher assistance in a Consumer Mathematics class.

Courses may also be offered through the use of electronic technology. In particular, interactive video systems are now available for delivering a variety of courses. Currently, the education service centers in Regions IV, IX, and XX have such systems with courses available to every part of the state.

These are a few possibilities for offering a variety of mathematics courses. School districts may be able to design other creative alternatives either individually or in cooperation with other districts and education service centers. Several suggestions for scheduling mathematics courses with only five, six, or seven sections follow:

Five Sections of Mathematics

Year A

- Fundamentals of Mathematics
- Pre-Algebra
- Algebra I
- Geometry/with Informal Geometry*
- Precalculus

Year B

- Consumer Mathematics/with Mathematics of Consumer Economics*
- Pre-Algebra
- Algebra I
- Algebra II
- Computer Mathematics I

Six Sections of Mathematics

Year A

- Fundamentals of Mathematics
- Pre-Algebra
- Algebra I
- Geometry
- Precalculus
- Informal Geometry

Year B

- Consumer Mathematics
- Pre-Algebra
- Algebra I
- Algebra II
- Computer Mathematics I
- Mathematics of Consumer Economics

Seven Sections of Mathematics (two plans)

Year A

- Fundamentals of Mathematics
- Pre-Algebra
- Algebra I
- Geometry
- Precalculus
- Informal Geometry
- Mathematics of Consumer Economics

Year B

- Consumer Mathematics
- Pre-Algebra
- Algebra I
- Algebra II
- Precalculus
- Informal Geometry
- Computer Mathematics I

---

*Two courses offered in one room

OR

Year A
- Fundamentals of Mathematics
- Consumer Mathematics
- Pre-Algebra
- Algebra I
- Geometry
- Precalculus
- Informal Geometry

Year B
- Fundamentals of Mathematics
- Consumer Mathematics
- Pre-Algebra
- Algebra I
- Algebra II
- Computer Mathematics I
- Mathematics of Consumer Economics

Summary of High School Mathematics Courses

The following descriptions of high school mathematics courses give an overview of key features of each course. "What Next?" lists the most appropriate courses for student to take next.

Teachers and administrators must accept as a serious responsibility the appropriate placement of students in high school courses, especially in Grade 9. School district policy should address specific entry criteria for high school courses, including advanced and remedial courses, to ensure that students are placed appropriately. Guidelines in the course descriptions that follow recommend placing students in remedial courses according to how close to grade level they perform. Placement criteria may include mastery of essential elements, achievement on standardized tests, performance on TEAMS, grades, and teacher judgment. Placement decisions should be monitored closely, and provisions should be available to move students to other courses whenever a placement error is determined.

As an option district personnel may wish to establish procedures to allow highly qualified students to enroll concurrently in two mathematics courses normally considered sequential. This practice should be used judiciously and only if the purpose is to allow the student to complete more mathematics in high school than would otherwise be possible.

Under State Board rules, districts have the option of allowing students to receive one-half unit of credit for a semester of any one-unit course. The district may decide whether it will award one or one-half unit for any course and whether or not half units of credit for different mathematics courses may be combined to satisfy graduation requirements.

Fundamentals of Mathematics (1 unit)

Purpose:
To review a wide variety of basic skills from elementary and junior high mathematics programs and basic objectives from the Texas Educational Assessment of Minimum Skills (TEAMS). The emphasis of the course is on applications and problem solving.

For Whom?
Students entering high school who are functioning two or more years below grade level based on mastery of essential elements.

Prerequisites:
None. Students who have received high school mathematics credit in any course may not enroll in Fundamentals of Mathematics. School districts may allow students to take Fundamentals of Mathematics after Consumer Mathematics if the courses are offered only in alternate years.

What Next?
Consumer Mathematics or Pre-Algebra.

Consumer Mathematics (1 unit)

Purpose:
To apply basic skills in a wide variety of consumer situations.

For Whom?
Students with some mastery of basic skills who are functioning below grade level based on mastery of the essential elements.

Prerequisites:
None. Students with high school credit for Algebra I may not enroll in Consumer Mathematics.

What Next?
Pre-Algebra.

Pre-Algebra (1 unit)

Purpose:
To reinforce seventh and eighth grade skills and to provide a firm foundation for success in Algebra I.

For Whom?
Students who are competent in basic skills but who have not yet mastered Grade 8 essential elements and who are not yet functioning at the abstract level needed for success in Algebra I.

Prerequisites:
None. Students who have received one full credit in Algebra I may not enroll in Pre-Algebra.

What Next?
Algebra I (strongly recommended), Informal Geometry, or Consumer Mathematics.

Informal Geometry (1 unit)

Purpose:
To present geometry concepts through applications rather than through formal proofs.

For Whom?
Students who do not need a college preparatory course but who are interested in continuing in mathematics after Pre-Algebra or Algebra I.

Prerequisites:
Pre-Algebra or Algebra I are strongly recommended. This course is not recommended for students in Grade 9 and should not be taken by students with one full credit in geometry.

What Next?
Algebra I, Algebra II, or Geometry (depending on individual course prerequisites and on the sequence determined by the school district); Computer Mathematics I or Mathematics of Consumer Economics, if Algebra I has been completed.

Algebra I (1 unit)

Purpose:
To provide a foundation for higher level mathematics courses.

For Whom?
Standard course for students functioning on grade level. Appropriate for students who have successfully completed Grade 8 essential elements.

Prerequisites:
Grade 8 mathematics (for Grade 9 students) or Pre-Algebra is recommended.

What Next?
Algebra II or Geometry (depending on district's sequence), Mathematics of Consumer Economics, Computer Mathematics I, or Informal Geometry.

Algebra II (1 unit) and Geometry (1 unit)

Purpose:
To provide a foundation for higher level mathematics courses

For Whom?
Students who have completed Algebra I and who wish to continue their study of mathematics.

Prerequisites:
Courses should be taken in the sequence determined by the district. Algebra I is a prerequisite for both courses.

What Next?
After completion of both courses, Precalculus or Trigonometry, Computer Mathematics I or II, Mathematics of Consumer Economics, Probability and Statistics, or an independent study course.

Trigonometry (½ unit)

Purpose:
To provide a foundation for higher level mathematics courses.

For Whom?
Students with appropriate prerequisites who do not have access to Precalculus.

Prerequisites:
Algebra II and Geometry.

What Next?
Elementary Analysis and/or Analytic Geometry, Computer Mathematics II, Mathematics of Consumer Economics, Probability and Statistics, or an independent study course.

Elementary Analysis (½ unit)

Purpose:
To provide a foundation for higher level mathematics courses.

For Whom?
Students with appropriate prerequisites who do not have access to Precalculus.

Prerequisites:
Trigonometry.

What Next?
Analytic Geometry, Calculus, Computer Mathematics II, Mathematics of Consumer Economics, Probability and Statistics, Advanced Mathematics for Business, or an independent study course.

Analytic Geometry (½ unit)

Purpose:
To provide a foundation for higher level mathematics courses.

For Whom?
Students with appropriate prerequisites who do not have access to Precalculus.

Prerequisites:
Trigonometry.

What Next?
Elementary Analysis, Calculus, Computer Mathematics II, Probability and Statistics, Advanced Mathematics for Business, or an independent study course.

Precalculus (½-1 unit)

Purpose:
To provide a foundation for higher level mathematics courses. (Combines Trigonometry, Elementary Analysis, and Analytic Geometry into one course.)

For Whom?
Students with appropriate prerequisites.

Prerequisites:
Algebra II and Geometry. Course is not recommended for students with credit in Trigonometry, Elementary Analysis, or Analytic Geometry.

What Next?
Calculus, Advanced Mathematics for Business, Mathematics of Consumer Economics, Computer Mathematics II, Probability and Statistics, or an independent study course.

Calculus (½-1 unit)

Purpose:
A college-level calculus course.

For Whom?
College-bound students who have completed appropriate prerequisites and who plan to enter a college program requiring a strong mathematics background.

Prerequisites:
Precalculus (or Trigonometry and either Elementary Analysis or Analytic Geometry).

What Next?
Advanced Mathematics for Business, Computer Mathematics II, Probability and Statistics, an independent study course, or a college mathematics course.

Advanced Mathematics for Business (½-1 unit)

Purpose:
To provide an academically rigorous course on business applications of advanced mathematics.

For Whom?
Academically able students wishing to enter business or management fields at the university level.

Prerequisite:
Precalculus (or Trigonometry and either Elementary Analysis or Analytic Geometry)

What Next?
Calculus, Computer Mathematics II, Probability and Statistics, or an independent study course.

Mathematics of Consumer Economics (½-1 unit)

Purpose:
A practical, applied mathematics course providing algebraic and consumer applications of fundamental skills.

For Whom?
Students who have completed Algebra I and who need an alternative to the more traditional mathematics sequence.

Prerequisite:
Algebra I.

What Next?
Algebra II or Geometry (depending on individual course prerequisites and district's sequence of courses), Computer Mathematics I, Informal Geometry.

Computer Mathematics I (½-1 unit)

Purpose:
To investigate problem-solving applications of the computer on topics specific to mathematics.

For Whom?
Students with an interest in computer applications of mathematics.

Prerequisite:
Algebra I.

What Next?
(May be taken concurrently with any mathematics course above Algebra I.) Algebra II or Geometry (depending on individual course prerequisites and on district's sequence of courses), Mathematics of Consumer Economics.

Computer Mathematics II (½-1 unit)

Purpose:
To investigate problem-solving applications of the computer on mathematics topics.

For Whom?
Students with an interest in computer applications of mathematics.

Prerequisites:
Algebra II (may be taken concurrently); Geometry also recommended.

What Next?
Geometry or Precalculus (depending on individual course prerequisites and on district's sequence of courses), Mathematics of Consumer Economics.

Probability and Statistics (½ unit)

Purpose:
To apply higher level mathematics skills in probability and statistics applications.

For Whom?
Academically able students who do not wish to pursue Calculus.

Prerequisites:
Algebra II.

What Next?
Precalculus (or Trigonometry and either Elementary Analysis or Analytic Geometry), Computer Mathematics II, or an independent study course.

Independent Study Courses (½ unit each): History of Mathematics, Survey of Mathematics, Number Theory, Linear Algebra, Linear Programming

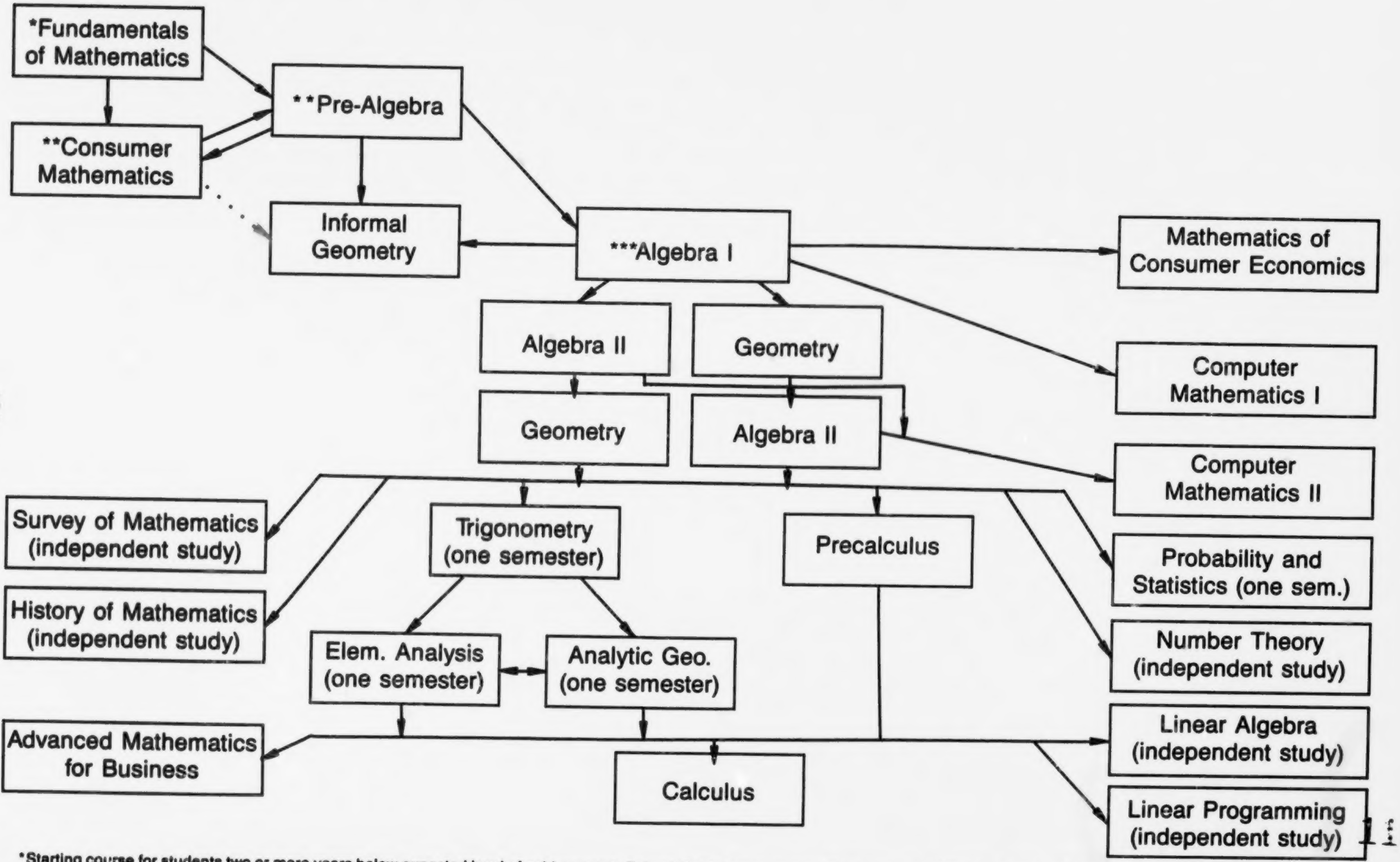
Purpose:
To provide a variety of topics in mathematics beyond the content of traditional courses.

For Whom?
Students with appropriate prerequisites whose needs may not be addressed in traditional courses.

Prerequisites:
Algebra I, Algebra II, and Geometry. For Linear Algebra and Linear Programming, Precalculus (or Trigonometry and either Elementary Analysis or Analytic Geometry), is also required.

What Next?
Independent study courses. These courses do not affect a student's progression through any sequence of mathematics courses as defined by school districts. These courses will be particularly important for students who are gifted in mathematics and who have completed requirements. Smaller districts may use independent study courses to offer advanced training to individual students who can work in small groups with little supervisory instruction.

Mathematics, Grades 9-12
Recommended Guidelines for Course Sequencing


*Fundamentals of Mathematics
**Pre-Algebra
**Consumer Mathematics
Informal Geometry
***Algebra I
Mathematics of Consumer Economics
Algebra II
Geometry
Computer Mathematics I
Geometry
Algebra II
Computer Mathematics II
Survey of Mathematics (independent study)
Trigonometry (one semester)
Precalculus
Probability and Statistics (one sem.)
History of Mathematics (independent study)
Elem. Analysis (one semester)
Analytic Geo. (one semester)
Number Theory (independent study)
Advanced Mathematics for Business
Linear Algebra (independent study)
Calculus
Linear Programming (independent study)

---

*Starting course for students two or more years below expected level of achievement. School districts may reverse the sequence of Fundamentals of Mathematics and Consumer Mathematics if courses are only offered in alternate years.

**Starting course for students one year below expected level of achievement.

***Starting course for students at expected level of achievement.

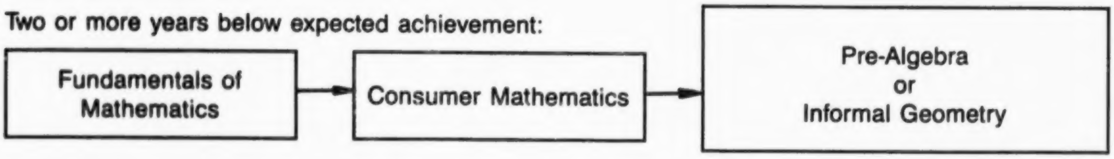
Do[illegible]rrow indicates an option that is not recommended but is available for distri[illegible]in making decisions for individual students.

BEST COPY AVAILABLE

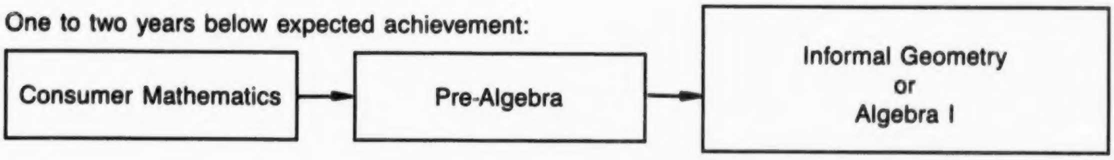
1986

Recommended Three-Year Sequences

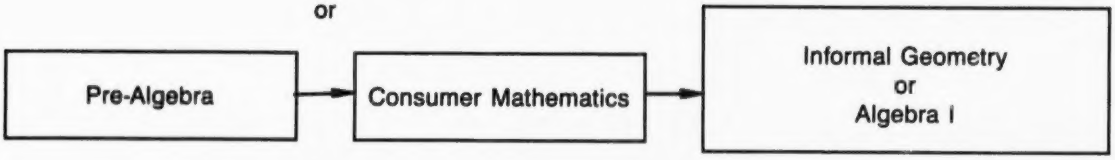
Two or more years below expected achievement:


Fundamentals of Mathematics
Consumer Mathematics
Pre-Algebra or Informal Geometry

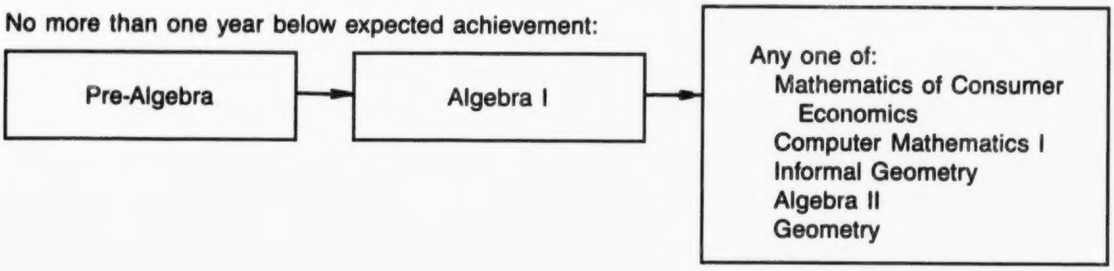
One to two years below expected achievement:


Consumer Mathematics
Pre-Algebra
Informal Geometry or Algebra I

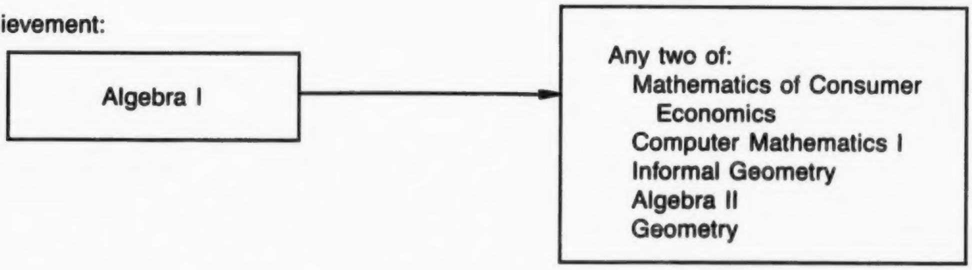
or


Pre-Algebra
Consumer Mathematics
Informal Geometry or Algebra I

No more than one year below expected achievement:


Pre-Algebra
Algebra I
Any one of:
Mathematics of Consumer Economics
Computer Mathematics I
Informal Geometry
Algebra II
Geometry

On-level achievement:


Algebra I
Any two of:
Mathematics of Consumer Economics
Computer Mathematics I
Informal Geometry
Algebra II
Geometry

1986

Honors Courses

High school honors mathematics courses should prepare students to pursue an advanced mathematics program in high school and at the college or university level. Courses must be submitted to the Texas Education Agency for approval six months prior to planned implementation. Honors courses must address an extension and/or expansion of the essential elements and must emphasize high level thinking skills, creative thinking, and cognitive concepts and processes. Provision must be made for a variety of learning styles, and students must have an opportunity to do independent and guided research. The courses listed below are appropriate to be submitted to the Agency for approval as honors courses:

Algebra I

If Algebra I is offered in Grade 8, there may not be a need for a high school honors section of Algebra I. Districts may want to consider the pros and cons of offering Algebra I in Grade 8 and also honors Algebra I in Grade 9 since the targeted students are largely the same for both. Since eighth grade Algebra I carries no high school credit, the school district can determine whether eighth grade Algebra I carries an honors designation. This designation would not affect the student's honors requirement in Grades 9-12. If Algebra I is offered as an honors course in high school, there must also be a nonhonors section available.

Algebra II

If Algebra II is offered as an honors course at the high school level, there must also be a nonhonors section available.

Geometry

If Geometry is to be offered as an honors course in high school, there must also be a nonhonors section available.

Precalculus or Trigonometry and either Elementary Analysis or Analytic Geometry

If an honors course is to be offered at the precalculus level, a nonhonors course option must be available. Any of these courses, Precalculus, Trigonometry, Elementary Analysis, Analytic Geometry, may be offered as honors or nonhonors options.

Calculus

Calculus is an honors course whenever it is offered, and no nonhonors option is needed. Calculus does not require approval as an honors course at the state level.

In addition to these courses, any other course identified in Chapter 75.152(b)(2) as a course appropriate for the advanced high school program may also be submitted as an honors course without the need for the district to offer a nonhonors option.

Special Student Populations and Mathematics

Introduction

Students comprising "special populations" include learners with a wide variety of characteristics. Some of these students are from migrant families. Some are gifted and talented. And some are limited in English proficiency. What these students share, despite their differences, is a need for alternative approaches to instruction in the classroom. For one reason or another, regular classroom strategies are sometimes inadequate for them.

Curriculum modifications for these identified students have much in common. They are strategies for good teaching that would benefit any student. For example, varying the method of instructing, adding personnel to achieve a one-to-one relationship, and changing the pace of presentation are all types of modifications teachers might use to meet individual student needs. For reteaching and remediation of a special education student, a teacher might employ the same strategy—breaking up a unit into smaller, sequential steps, for example—that might also meet a migrant student's special language needs. For this reason, repetition necessarily occurs in the discussion of modifications in the following sections.

Some students need a concentrated effort that uses extensively one or more curriculum modifications specific to their needs. These students may have a handicap, may be in difficult economic situations, or may experience language deficiencies or chronic underachievement. Developing modifications tailored to the specific situation of these students challenges any instructional staff. Administrators, curriculum supervisors, and classroom instructors may find it helpful to use this chapter, Special Student Populations and Mathematics, as a pool of suggestions of teaching practices widely held to be effective for most students. They can also use individual sections as references for specific suggestions as they identify and serve students with individual needs.

Goals

A major task of an education program is to provide all students with appropriate opportunities to grow physically, emotionally, aesthetically, and intellectually to the full extent of their abilities. Prior to the curriculum reform of the mid-1980s, some students received remedial instruction which, in many cases, was isolated, fragmented, and unrelated to the regular instructional program. Consequently, many students were unable to maintain steady progress when they returned to the regular program of instruction.

The state basic curriculum now requires a process that ensures a well-balanced, single curriculum of instruction for all students regardless of special need or condition. Instruction for students with special needs includes the same essential elements as the instruction given general education students. Students with special instructional needs may encompass a wide range of academic abilities and language proficiency levels in English as well as in a language other than English.

As soon as teachers observe that student performance is inadequate, they should investigate various special programs available. They should initially consider whether use of curriculum modification strategies might help the student in the regular classroom. Even if students are progressing at a normal pace, strategies to vary methods of presentation, utilize additional personnel, and modify forms of assessment can make teaching and learning more effective.

Special program personnel and regular instructional personnel are responsible jointly for the cooperative delivery of effective instruction in the essential elements. School district programs for students identified as having special needs should modify the method of instruction, pacing, and materials, as appropriate, to provide full opportunity for learning the essential elements.

Special Education

Depending on how a handicap affects a student's ability to master the curriculum, handicapped students may be able to fulfill the same mathematics requirements as their nonhandicapped peers. They may, however, require modified content, mastery criteria, testing methods, instructional methods, materials, setting, and pacing. The individual educational plan (IEP) developed by the admission, review, and dismissal (ARD) committee is the district's official record of services it provides to ensure the handicapped student an appropriate education.

The IEP lists any modifications necessary for satisfactory progress whether the handicapped student is placed in regular or special education classes. Although most handicapped students in regular classes have normal intelligence, they may need modified methods of instruction to demonstrate satisfactory progress. Generally, the inability to perform without modification can be attributed to one or more of the following limitations:

- physical—affects motor coordination, fatigue, causes frequent absences
- sensory—affects visual or auditory reception and/or expression
- cognitive—affects receptive and/or expressive communication skills
- emotional—affects behavioral and social skills

The IEP may also direct the regular teacher regarding the specific modifications that need to be made for the student in the regular classroom. Local district procedures direct how the modifications are communicated to school personnel responsible for implementation and how the modified instruction is evaluated. Unless otherwise stated in the IEP, the regular teacher is responsible for teaching the essential elements to the handicapped student in the regular classroom.

General Principles of Modification

Special education instruction can reinforce, follow-up, or supplement basic instruction. For example, the special education teacher can provide:

- alternative methods to present the materials to be learned
- supportive instruction and reinforcement using the same materials as regular teachers
- special instructional materials or media to reinforce concepts presented in regular classes
- adaptive methods to assess a student's knowledge of the subject area or essential elements in the regular classroom
- assistance in determining and demonstrating appropriate behavior in the regular classroom

Modification of Evaluation

When the district's regular mastery standards are inappropriate for a handicapped student, the ARD committee specifies in the IEP the knowledge and skills the handicapped student should attain as an indication of progress or mastery.

Testing methods must allow the student to demonstrate knowledge and skills in mathematics regardless of the disability. For example, the evaluation of mathematics applications in word problems for a handicapped student with severe reading disabilities should not be based on test methods that require average reading and writing skills.

Modification of Materials

Special education materials are available to help regular teachers carry out modifications required in the IEP. Visually handicapped and learning disabled students may be given tape recorded textbooks and large print or braille materials.

Modification of Methods

When handicapped students are placed in regular classes for mathematics, the regular teacher can expect to be given specific information about the individual student's skill levels and modifications needed. In general, handicapped students' disabilities may result in their experiencing difficulty with mathematics in two major areas:

- reasoning
- calculation

The teacher should evaluate the students' performance in both of these related areas to see how difficulty with reasoning and calculation affect performance in the specific course. Problems with reasoning in mathematics may result in difficulty in grasping concepts, facts, laws, and operations, and, consequently, in formulating appropriate solutions to problems. Some indicators that may be obtained when the handicap affects the students' mathematics reasoning and calculating skills are that students do not:

- organize a task to facilitate its completion
- make appropriate inferences from data
- appropriately sequence occurrences or events
- make reasoned choices but guess at answers
- indicate knowledge of spatial relationships such as directions and location
- discriminate differences or similarities in size and shape
- employ problem-solving strategies; often use random, trial-and-error approaches to tasks
- correctly write the order of written digits, place-value notation, etc.
- use cues correctly
- display accurate logic when computing
- accurately understand process signs such as "+" and "−" and other vocabulary such as "into," "left," "dozen," etc.
- remember numerals or words needed for arithmetical operations

Because of the additional difficulty in reading, writing, and language, most handicapped students will require more time to complete mathematics assignments and tests than their nonhandicapped peers. Students may not finish the assignment because they spend too much time rereading, they are unsure of the importance of what they have read, or they tire and skip important information necessary to complete the assignment accurately.

Nonreading experiences such as team projects, manipulative activities, audiovisual aids, and demonstrations enable handicapped students to learn mathematics concepts through active participation. Based on the teacher's observation of the handicapped student's needs, teachers can make numerous adjustments in the regular classroom.

The teacher may:

- clarify course assignments for students
    - repeat instruction, provide more detailed instructions
    - provide course syllabus or outline
    - use individual contracts with specific requirements
    - use individual learning packages with clearly stated objectives
    - use progress charts and informal individual feedback interviews
    - give instructions through several channels such as written and oral
    - provide materials that are programmed or self-checking
    - brief students on key points before starting an assignment
    - allow students to tape record assignments
    - provide correctly completed examples and sample problems
    - group similar items or problems
- adapt regular assignments
    - require fewer correct responses for a specific grade
    - allow more time for regular assignments
    - reduce the length of regular assignments
    - break assignment into a series of smaller assignments or mini-series
    - underline or outline major points in the regular assignments; ask leading questions
    - have students complete problems the teacher starts
    - space problems far apart on the page
    - include pictures and illustrations whenever possible
    - color code important words or symbols such as plus, minus, exponent, and brackets
    - provide graph paper for students to keep columns of numbers and problems straight
- use alternative materials to supplement the regular text and workbook:
    - use different format materials to teach the same content such as puzzles, games, tapes, and pictures
    - combine high-interest, high-motivation materials with drill materials
    - borrow, share, and acquire remediation or reinforcement materials from other grade-level teachers and from the special education materials collection
    - provide study aids such as hints, cue cards, spelling lists, and guides
    - provide computational aids such as counting blocks, number lines, abacus, and calculators
- use alternatives to written and reading assignments
    - use hands-on activities, physical assignments, and performances
    - use oral presentations, reports, and projects

- provide alternative methods to complete assignments
    - use peer tutors to assist handicapped students in practice and drill activities
    - provide teacher aide assistance
    - use group assignments such as projects and panels
    - have other students read the assignments to poor readers and write down the directions for homework assignments
- modify examination procedures
    - change number of solutions required
    - allow open book exams
    - allow more time for or reduce the length of regular exams
    - write out hints or reminders on the test
    - give the exam orally or allow students to use manipulatives
    - reduce the reading level or reword story problems on the test by paraphrasing; substitute regular problem format for story problems or tape record story problems
    - use fill-in responses for word or definition problems with a space provided for each letter of the answer
    - write down test item responses
    - give take-home tests
    - substitute assignments for tests
    - allow a peer, teacher aide, or volunteer to administer tests individually
    - provide lists of multiple choice test responses
    - grade tests according to number of items correct out of number attempted
    - allow student to make a spiral notebook word bank that references vocabulary and terms frequently used in class
- use projects that demonstrate student knowledge without demonstrating reading skills such as
    - posters and charts
    - drawings and illustrations
    - flannelboard designs
    - shoebox panoramas or other models and constructions
    - collections

Gifted and Talented Education

All educational programs should be developed to meet the needs of individual learners, and mathematics programs for gifted students are no exception. An understanding of the needs and characteristics of gifted students is the first step in designing an appropriate curriculum for them.

Meeting specific student needs is one reason why effective schools stress differentiated curriculum for gifted learners. A differentiated curriculum is not more of the same but rather a curriculum that accelerates, expands, and enriches the three basic elements of the regular curriculum: content, process, product. An appropriate curriculum for gifted students includes not only accelerated or intense content matter. Rather, it develops a complex and sophisticated product that requires adept use of process skills.

Many educators contend that the development of higher level thinking skills, of independent study skills, and of research skills should be included in the mathematics curriculum for all students. Without question, this is true. However, because gifted students generally grasp the basic skills more rapidly than other students, they should be able to spend more of their instructional time on these advanced skills.

The mathematics curriculum for gifted students must be based on the curriculum for all students because all students need the basic skills the regular curriculum emphasizes. However, because students should never have to wait on instruction but rather should be able to progress at their own pace, gifted students will need appropriate curriculum developed to meet their special needs. Mathematics instructors should not require gifted students to complete 20 additional problems because they rapidly completed the 10 problems assigned. Instead, efforts should be made to develop more complex mathematical skills using the basic skills already mastered.

Chapter 75 Board rules provide school districts with many alternatives to instruction that are excellent vehicles for courses for gifted students. Honors courses provide one alternative; however, unless the curriculum of the honors courses is differentiated for gifted learners, these courses ordinarily will not be appropriate to meet their needs.

Experimental courses and magnet programs can be good alternatives for gifted students and are promising options for students in larger districts. Correspondence courses are an alternative that might be useful in smaller districts that do not have enough students to form specific classes. Advanced placement may also be an option for mathematically gifted students who can work independently.

The instructional program for gifted students should begin as early as possible, preferably in prekindergarten or kindergarten. Gifted students arrive at school gifted and should not have to wait until Grade 1 to receive an education commensurate with their abilities.

Finally, the mathematics curriculum for gifted students must contain a framework for coordination of the entire program and a scope and sequence for content, skills, and products. Because many students are served in pull-out programs or in resource rooms, it is easy for gaps to develop in their learning if no effort is made to coordinate the gifted with the regular program.

Because education for the gifted is a relatively new area for most districts, additional assistance might be needed in curriculum development. General information about gifted students may be obtained from the Division of Gifted and Talented education, and specific mathematics information may be obtained from the mathematics consultants in the Division of Curriculum Development at the Agency.

Special Programs

Compensatory Education

Compensatory instruction is directed toward all students who are having academic difficulty in mathematics, English language arts, reading, science, and/or social studies. Students who may require remedial or compensatory instruction include migrant students, students who speak a language other than English, and prekindergarten or kindergarten children who have special developmental needs. Compensatory instruction should extend and reinforce the regular program of instruction.

Compensatory education should be:

- based on each student's functional instructional level
- modified as necessary to accommodate methodologies, pacing, and materials
- designed to teach the essential elements

There are many reasons why students may need special assistance to master the mathematics curriculum. Some students may need assistance with a single concept or skill only, while others may be functioning significantly below the age or grade level of their peers. When it is apparent to the mathematics teacher that a student is not succeeding in the instruction provided, modifications in learning styles and materials as well as other changes should be initiated.

General Principles of Modification

In providing compensatory education, teachers must:

- assess language proficiency, if necessary, to determine program placement in the appropriate language for instruction
- determine instructional placement in materials to ensure that instruction is provided at the ability level of the student
- identify specific skill needs to ensure that instruction is directed toward specific skill and concept deficits
- identify specific learning modalities and styles
- identify alternate instructional strategies

Modification of Method, Personnel and Setting

Teachers are encouraged to:

- use small group instruction for students with same skill needs
- limit drill and practice activities to shorter periods of time
- ensure close monitoring of practice activities to prevent repetition of errors
- place more emphasis on problem solving and development of concepts
- use concrete activities to teach skills and concepts

- reinforce reading skills in which students have been diagnosed as deficient using activities that promote:
    - understanding of mathematics vocabulary
    - reading graphs
    - banking and shopping skills
    - following directions
    - reading and understanding word problems
    - drawing conclusions in word problems
- use cross-age tutors, adult tutors, and peer tutors for one-on-one reinforcement
- provide coaches such as parents and teachers to serve as mentors
- provide computer-assisted instruction for drill and practice activities

Modification of Materials

In providing materials for compensatory education, teachers may:

- secure multilevel materials that incorporate the essential elements for the unit or course of study
- develop or obtain multimedia materials and equipment to accommodate different learning styles
- provide manipulative and tactile materials for concept and skill development

Bilingual Education

Mathematics for Students Limited in English Proficiency

To ensure a quality mathematics preparation for limited English proficient (LEP) students, administrators must provide a program having all the salient characteristics of a quality mathematics program for native speakers. For native English speakers, a quality program adds to basic literacy skills expanded meanings of language. It builds vocabulary and teaches word recognition and comprehension techniques. Therefore, English-speaking students have the developmental skills upon which to establish the additional competencies needed for successful performance in the mathematics program.

On the other hand, the LEP student who is struggling to speak, read, and comprehend the English language may have few, if any, of the preskills necessary for academic success in mathematics. The reading requirements are often extensive and unrealistic for these students, and their comprehension of the abstract concepts basic to mathematics is usually minimal. Success in mastering mathematics concepts is possible only to the extent that the LEP students have acquired the basic oral proficiency and reading skills, specialized abilities, and knowledge demanded in mathematics.

Fortunately for many LEP students, numbers and ideas based directly on numbers are universal. Therefore, for many LEP students who do not read English well, the solving of mathematics problems presented in familiar terms comes quite easily. Students will have more difficulty, however, when they are asked to learn, understand, and apply mathematics concepts based on a reading. These difficulties result from the fact that:

- the concept and the process may be known to the student but the ability to express the knowledge is minimal
- when problems are posed in written form and not in Arabic numerals, students are unable to read the problem correctly
- mathematics skills are inadequate, and comprehension is impossible without reteaching and remediation

When LEP students are acquiring English language skills and simultaneously receiving mathematics instruction in printed English, they need to concentrate on developing these basic competencies:

- word recognition techniques to build reading vocabulary
- comprehension of phrases, sentences, and expressions peculiar to mathematics vocabulary such as abbreviations and technical terms
- ability to organize and sequence mathematical information
- ability to follow specific instructions on how to solve problems

A LEP student cannot realistically be expected to learn to understand, speak, read, and write a second language simultaneously with acquiring comprehension of the specialized language of mathematics. Therefore, two relevant and basic questions a teacher specializing in mathematics must consider are:

- How much prior skill and knowledge of mathematics does the student possess?
- How effectively is the student able to read the mathematics textbook?

Unless the student has significant prior skill and knowledge and can effectively read the text, modifications of the mathematics curriculum will be necessary.

To teach mathematics to LEP students, two distinct programs, bilingual education and English as a second language (ESL), may be used as modified approaches to the coursework. A bilingual education program makes use of a LEP student's native language as well as some English to teach mathematics. An ESL program uses the second language, English, as the medium of instruction along with adaptive methodologies.

For LEP students to benefit from instruction in mathematics, an assessment of the students' language abilities is needed to determine which language should convey the contents of the course. For example, LEP students who understand and speak English with some adequacy but who have not yet learned to read and write well in English will succeed more rapidly in learning mathematics concepts through oral and visual methods than through reading assignments.

When students are literate only in their native language, subject matter should be made available in the written form of that language. Appropriate language conveyance will enable LEP students to keep pace with their English-speaking peers in course content.

General Principles of Modification

To meet the instructional needs of the LEP student through bilingual education and ESL programs, districts may modify their programs for learning English and for mastering the mathematics essential elements required.

The program as modified in a dual language program should begin:

- to emphasize and reinforce the concepts and ideas of mathematics as the students learn English
- to emphasize acquisition of basic mathematics skills
- to foster the thinking process
- to stimulate continuous reorganization of previously learned ideas
- to encourage further investigation of the information presented
- to support multiple learning styles

While modification of the instructional program involves altering the language in which the content is conveyed, altering the scope of the curriculum would not occur in a bilingual education program.

In an ESL program, the sequence in which the essential elements are presented may be modified to accommodate the students' progress in acquiring English language skills. Emphasis on oral and visual stimuli aid students in comprehending concepts English-speaking students develop through written material.

Other instructional strategies may be designed:

- to help students build card files and glossaries on needed mathematics vocabulary
- to show the same information through a variety of different graphs and visuals
- to encourage students to underline key words or important facts in their written assignments
- to encourage categorizing of mathematical words into meaningful groups
- to pair students for team learning
- to teach English expressions for mathematical operations such as "square," "add," and "multiply"
- to teach measurement terms through use of the actual instruments and devices whenever possible
- to teach the names of currency, coins, and monetary units
- to teach words that indicate quantitative relationships such as "more," "less," "larger," and "twice as many"
- to teach prefixes peculiar to the language of mathematics such as "bi-," "deci-," "centi-," and "kilo-"

Modifications may also include the teaming of bilingual or ESL certified instructors with personnel on emergency teaching permits to develop supplementary programs for LEP students. Parent volunteers and paraprofessionals can work with regular instructional personnel cooperatively to deliver the needed programs in accordance with the State Board rules.

Modification of Pacing and Teaching Strategies

The LEP students' program must be paced according to language and academic abilities and should employ teaching strategies that take these students' unique needs into consideration. Course selection and choice of textbooks should be carefully supervised by teachers, counselors and administrators.

In an ESL program, pacing modifications concentrate on vocabulary and concept development; in a bilingual program, dual language instruction concentrates on a process approach to the content area. In addition, teachers may:

- explain special vocabulary terms in words known to the students
- write instructions and problems using shorter and less complex sentences
- prepare a sequence of ordinal numbers and identify the ordinal position of each one by writing the appropriate numerical symbol and word; for example, "lst-first"
- assign short homework tasks that require reading
- limit the number of problems that must be worked
- emphasize special mathematical meanings of words commonly used in English
- tape problems for independent listening assignments
- deemphasize speed and emphasize accuracy of work
- give problems requiring several solutions

Modification of Materials

All materials, whether state-adopted textbooks in Spanish, teacher-made lessons, or district-developed aids to instruction, should be modified to meet the students' academic needs. Administrators and curriculum supervisors might provide support for classroom instructors to:

- provide numerous pictures to illustrate new words
- offer a variety of reference materials at the student's instructional level for independent use
- keep a variety of number games to be played by pairs of students or small groups
- maintain a library of supplementary books and workbooks written in simple English which offer additional illustrations for problems
- keep listening tapes on mathematical problems for individual assignments
- help students prepare a card file of number words and problems
- have students prepare glossaries of mathematics terms
- encourage the use of diagrams and drawings as aids to identifying concepts and seeing relationships
- support reading in mathematics by having films, records, filmstrips, and other materials which may be used independently or in small groups

Districts may find it helpful to consult the published readability levels of state-adopted textbooks as they select materials that will be appropriate for LEP and ESL students.

Migrant Education

Modifications appropriate for gifted and talented, compensatory, and bilingual students are also frequently appropriate for migrant students. While some migrant students will be limited in English proficiency (LEP), some will be in need of compensatory education strategies, and some will need curriculum enrichment. The unique characteristics of migrant students may necessitate additional modification to keep these students' performance on grade level.

Migrant students move and live primarily in agricultural settings. High mobility may cause the student to have received piecemeal instruction from several schools. Migrant students may attend school part-time, from six to seven months; or they may attend school for the full year but still have gaps in their education due to high mobility.

A useful source from which the district may extract vital academic information concerning the mobility patterns of individual migrant students is the educational record of the Migrant Student Record Transfer System (MSRTS). This record reports the three latest skills mastered in mathematics and the three latest skills under study by the migrant elementary student in Grades 1-8.

Texas Educational Assessment of Minimum Skills (TEAMS) scores are also listed in the record. For the migrant secondary student in Grades 9-12, the record reports mathematics courses taken, partial work for incomplete courses, and TEAMS scores. Where migrant counselors are available, they provide a useful resource in helping to place migrant students in the appropriate mathematics coursework in new schools.

Continuity of instruction can be accomplished through the use of MSRTS. If a migrant student withdraws, for example, the school district should promptly transmit information about mathematics skills mastered or under study to the terminal site at the local education service center (ESC). The ESC will then transmit the data to a national data bank in Little Rock, Arkansas. When the student reenrolls, the information will be readily available to the new district for instructional planning. Students whose schooling has been interrupted will benefit greatly from highly structured time-on-task activities and from a lower student/teacher ratio.

Teachers of migrant students should be aware that:

- the students' skills may be based in another language
- assessment for proper language to convey course content is crucial to migrant students
- student achievement may be difficult to evaluate accurately until language assessment is complete

Two modifications are basic to ensuring provision of a well-balanced curriculum for the migrant student:

- provision of continuity of instruction
- assurance of intensive instruction

General Principles of Modification

The following suggestions are recommended for teachers of migrant students:

- Assess the student's understanding of mathematic skills through informal testing. If remediation is needed in certain skills or areas, tailor instruction to cover those skills first while incorporating other mathematical on-grade-level concepts that the student can comprehend.
- Check the readability level of the textbook and modify vocabulary if necessary. If, for example, a problem reads, "Find the perimeters of the triangle by adding the sums of the sides," the instructor can state or write more simply, "Find the distance around the triangle by adding the length of each side."
- Select and highlight the most important mathematic concepts that will address the essential elements.
- Break complex operations into small steps and teach each step separately.
- Prepare guides that focus on common errors in the material and develop a problem-solving procedure to help detect errors.
- Adjust teaching to learning styles.
- For elementary students, build independent units that can be rearranged as needed to cover areas that have not been taught.
- Establish a buddy system for completion of assignments. This is especially effective for migrants who often feel uncomfortable in a new environment.
- Use field trips to further expose or develop mathematical concepts.
- Listen to students' vocabularies and use their words to explain concepts and state problems.
- Incorporate as much language development as possible in the lessons.
- Use the migrant experience to develop exercises such as problems dealing with distances and problems dealing with piecework rates or hourly wages.
- As instruction progresses, assist students to apply, synthesize, and evaluate skills and concepts they have already mastered.
- Make generous use of manipulatives.
- Be certain partial work done in other districts is completed.
- Provide opportunities for small-group instruction and modules that are self-paced.
- Provide mentors for gifted migrant students who show high ability and interest in particular mathematics fields.

Modification of Pacing

Teachers are encouraged to:

- incorporate an ongoing evaluation system of mathematics skills into the curriculum so that the migrant student is moved to the next mathematical concept as soon as skills are mastered
- prepare independent modules which the students may complete at their own pace; if reading is a problem, consider taping instructions
- use small groups and learning centers to differentiate instruction and allow for individual pacing
- to supplement instruction, use tutors such as peer tutors, parents, teachers, teacher aides, and others working in small groups or on a one-to-one basis
- establish extra classes during the school day, in the evening, on weekends, and during the summer
- provide computer-aided instruction for remedial and enrichment purposes

The students' migrant experience should be assessed to determine facets that may help them to grasp and master certain mathematics concepts more rapidly. For example, the students' experience in the fields may help them understand weights and measures. Secondary students may be able to use the experience of traveling to understand how to calculate budgets and estimate mileage.

Pacing is especially important for gifted migrant students. Their experience should form a foundation from which an understanding of the essential elements can be rapidly expanded. These students can be led to understand how to add, multiply, and divide complex numbers by relating these abstract mathematical concepts to the concepts of production and profit which they have experienced dealing with crop harvesting. The student can be asked to question how volume affects cost and profit and how a profit margin affects prices of a commodity and pay scales.

Modification of Materials

In providing materials for migrant students, teachers may:

- use supplementary materials with suitable reading levels whenever appropriate
- use the total curriculum materials to reinforce mathematics essential elements
- modify the materials to demonstrate mastery. For example, manipulation of materials, rather than a paper-and-pencil test, can show whether a concept has been mastered.
- maintain materials for independent projects that will enhance problem-solving skills

Effectively teaching mathematics to migrant students is a challenge to many Texas districts. Thorough, careful analysis and diagnosis are needed to determine the exact nature of their needs. The district must assess whether migrant needs are due to:

- lack of experience
- lack of time on task
- need for remediation

An accurate assessment will have implications for the content of the curriculum, methods of instruction, and the direction of the program designed to meet migrant students' needs.

Appendices

A Position Statement

Professional Development Programs for Teachers of Mathematics

By the National Council of Teachers of Mathematics, 1985

Teachers of mathematics, like all professionals, require ongoing and cumulative professional development programs that enhance and maintain their teaching skills and knowledge. Because mathematics and education are disciplines that grow and change, teachers cannot depend on what they learned as undergraduates to carry them through their entire careers. Findings of research continually increase our understanding of teaching and learning. Further, social and technological changes increase the average citizen's need to understand and use mathematics. These forces demand reconsideration of the content and methods of mathematics instruction.

Curricular and instructional changes, however, do not occur automatically. The extent to which new ideas and techniques are integrated with current classroom practices depends on teachers' knowledge, motivation, and commitment to continued professional growth. The improvement of mathematics programs depends on well-prepared and well-informed teachers.

Such changes and improvement require teachers to have opportunities for high-quality professional development. The provision of these opportunities, which should maintain, enrich, and improve the skills and abilities that teachers need to serve their students best, is the shared responsibility of districts, schools, and individual teachers.

To help promote high-quality classroom instruction in mathematics, the National Council of Teachers of Mathematics encourages and supports the development and implementation of comprehensive professional development programs. The Council recommends that such programs be developed in accord with the following guidelines:

1. Professional development programs for teachers of mathematics should be based on a strong commitment to professional growth.
    a. An appropriate person should be responsible and accountable for the professional development of the teachers.
    b. Sufficient time should be allocated for individuals to assess the needs, plan activities, lead or participate in programs, and evaluate outcomes.
    c. Sufficient funds should be available to support professional development programs and ensure teachers' participation in them.
2. Professional development programs for teachers of mathematics should be carefully planned.
    a. Clear objectives should be established.
    b. The programs should improve students' learning experiences by improving the skills and knowledge of their teachers.
    c. Those whom the programs are designed to assist should contribute significantly in planning the programs.
    d. Extensive assessment of individual and collective needs should serve as bases of the programs.
    e. Current concerns and issues in mathematics education should be reflected in the content of the programs.
    f. The programs should be ongoing and cumulative.

3. Professional development programs for teachers of mathematics should recognize individual differences.
    a. Varied formats, including workshops, conferences, institutes, courses, and in-school discussion sessions, should be used.
    b. Programs should be tailored to meet the needs of teachers whose knowledge, skills, and experiences are diverse.
4. Professional development programs for teachers of mathematics should be effectively conducted and should include the following features:
    a. A blending of mathematical content and effective pedagogy
    b. Active participation of teachers
    c. Attention to the concrete, day-to-day problems of teachers
    d. An integration of theory and practical applications
    e. Communication of objectives to participants
    f. Opportunities for teachers to practice new skills and techniques in the classroom.
    g. Incorporation of support and follow-up activities
5. Professional development programs for teachers of mathematics should be systematically evaluated, with attention to these issues:
    a. Determining whether the needs they are designed to meet have been satisfied.
    b. Using the results from the evaluation to improve and develop future programs

Excellent sources of assistance in implementing the guidelines listed above include the meetings, conferences, and publications of the National Council of Teachers of Mathematics and its local, state, or provincial affiliated groups.

A Position Statement

Calculators in the Mathematics Classroom

By the National Council of Teachers of Mathematics, 1986

The National Council of Teachers of Mathematics recommends the integration of the calculator into the school mathematics program at all grade levels in classwork, homework, and evaluation. Although extensively used in society, calculators are used far less in schools, where they could free large amounts of the time that students currently spend practicing computation. The time gained should be spent helping students to understand mathematics, to develop reasoning and problem-solving strategies, and, in general, to use and apply mathematics.

At each grade level, every student should be taught how and when to use the calculator. To use calculators effectively, students must be able to estimate and to judge the reasonableness of results. Consequently, an understanding of operations and a knowledge of basic facts are as important as ever. The evaluation of student understanding of mathematical concepts and their application, including standardized tests, should be designed to allow the use of the calculator.

The National Council of Teachers of Mathematics recommends that all students use calculators to:

- concentrate on the problem-solving process rather than on the calculations associated with problems
- gain access to mathematics beyond the students' level of computational skills
- explore, develop, and reinforce concepts including estimation, computation, approximation, and properties
- experiment with mathematical ideas and discover patterns
- perform those tedious computations that arise when working with real data in problem-solving situations

The National Council of Teachers of Mathematics recommends that publishers, authors, and test writers integrate the use of the calculator into their mathematics materials at all grade levels.

Selected References: Encouraging Full Participation in Mathematics

All students—including minorities and girls—should have the opportunity and the encouragement to fully participate in the study of mathematics. The following materials should help educators see that every student takes all the mathematics courses he or she will need to prepare for college and the world of work. A number of the publications are written for students.

Beane, DeAnna Banks, ed. *Mathematics and Science: Critical Filters for the Future of Minority Students.* The Mid-Atlantic Center for Race Equity, Washington, D.C., 1985. (pamphlet)

Bingham, Mindy; Edmondson, Judy; and Stryker, Sandy. *Choices: A Teen Woman's Journal for Self-awareness and Personal Planning.* Santa Barbara, CA: Advocacy Press, 1983. (book)

The College Board. *Academic Preparation for College: What Students Need to Know and Be Able to Do.* NY: The College Board, 1983. (book)

__________. *Academic Preparation for the World of Work.* NY: The College Board, 1984. (booklet)

Coordinating Board, Texas College and University System. *Goals for College Success: A Practical Reference for College Preparation,* 1982.

Iker, Sam. "A Math Answer for Women." *Mosaic* (National Science Foundation), *11*(3), May/June 1980, pp. 39-45. (article)

Kansky, Bob, and Olson, Melfried. *Mathematical Preparation versus Career Aspirations. A Study of Wyomings' 1978 High School Seniors.* Laramie, WY: University of Wyoming, 1979. (Report); ERIC Document Reproduction Service No. ED 168 263

Manitoba Education. The series *Mathematics: Who Needs It?,* Department of Education, Manitoba, Canada, 1986 (four pamphlets and a poster)

Mathematical Association of America. *The Math in High School You'll Need for College.* Washington, DC: MAA, 1985. (brochure)

Mathematical Association of America. *Professional Opportunities in the Mathematical Sciences.* Washington, DC: MAA, 1983. (booklet)

Mathematical Association of America. *You Will Need Math: Information for High School Students.* Washington, DC: MAA, 1985. (brochure)

Maurer, Stephen B. "College Entrance Mathematics in the Year 2000." *Mathematics Teacher, 77*(6), September 1984, pp. 422-428. (article)

Mitchell, Charles E. "The Importance of Three Years of High School Mathematics." *Mathematics Teacher, 77*(7), October, 1984, pp. 510-513. (article)

National Council of Teachers of Mathematics. *Handbook for Conducting Equity Activities in Mathematics Education,* Reston, Virginia, 1984. (book)

The National Science Board Commission on Precollege Education in Mathematics, Science and Technology. *Educating Americans for the 21st Century,* 1983.

Oklahoma State Department of Education. *The College Connection,* Planning for college sucess while still in high school, Oklahoma City, Oklahoma 73105, 1983.

Olson, Melfried, and Kansky, Bob. "Mathematical Preparation versus Career Aspirations: Sex-Related Differences Among College-Bound Wyoming High School Seniors." *Journal for Research in Mathematics Education, 12*(5), November 1981, pp. 375-379. (article)

Sells, Lucy. "The Critical Role of Elementary School Mathematics in Equalizing Opportunity." *Arithmetic Teacher, 29*(1), September 1981, pp. 44-45. (article)

__________. *High School Math as the Critical Filter in the Job Market.* Berkeley, CA: Author, 1973. (Report) (ERIC Document Reproduction Service No. ED 080 351)

__________. "Improving Educational Opportunity in Essential Mathematical Skills." *The School Administrator* (American Association of School Administrators), *36*(11), December, 1979, pp. 22-23. (article)

__________. "Leverage for Equal Opportunity Through Mastery of Mathematics." In Sheila Humphreys, (Ed.) *Women and Minorities in Science: Strategies for Increasing Participation.* Boulder, CO: Westview Press for the American Association for the Advancement of Science, 1982, pp. 7-26. (article)

__________. "Mathematics—A Critical Filter." *The Science Teacher, 45*(2), February, 1978, pp. 28-29. (article)

__________. "Mathematics: The Invisible Filter." *Engineering Education* (American Society for Engineering Education), *70*(4), January, 1980, pp. 340-344. (article)

__________. "Multiple Levels for Change." In Judith E. Jacobs (Ed.) *Perspectives on Women and Mathematics.* Columbus, OH: ERIC Clearinghouse for Science, Mathematics and Environmental Education, 1978, pp. 137-142. (article)

Tomhave, William K. "Motivating Students to Improve Their Math Preparations." *NASSP Bulletin* (National Association of Secondary School Principals), *64*(483), October 1985, pp. 89-91. (article)

Wells, J. Norman. "Keep Your Options Open." *The American Mathematical Monthly, 89*(8), pp. 589-590. (article)

1986

Suggested Materials for Teaching Informal Geometry

In addition to state-adopted textbooks, a wealth of nontextbook information and teaching resources is available to help teach Informal Geometry. Some of these sources are listed below. This list is not intended to be a recommendation or endorsement, nor can it be considered comprehensive. Rather, it provides a starting point for collecting resources.

The books listed are for teacher reference and are not necessarily recommended for student use.

Abbot, Edwin A. *Flatland.* Dover, 1952.

Aldridge, Adele. *Once I Was A Square.* Magic Circle Press, 1974.

Bates, Thomas and Long, Calvin, eds. *Mathematics in Modules.* Rand McNally and Co., 1976.

Beard, Col. R. S. *Patterns in Space.* Creative Publications, Inc., 1973.

Bennett, Albert and Nelson, Leonard. *Mathematics: An Activity Approach.* Allyn and Bacon, 1979.

Escher, M. C. *The Graphic Work of M. C. Escher.* Meredith, 1967.

Eves, Howard. *A Survey of Geometry, Volume One.* Allyn and Bacon, Inc., 1963.

Goldberg, Steven A. *Pholdit.* Billiken Publications, Inc., 1972.

Hick, Kim. *Geo-Dynamics.* Crystal Reflections, 1978.

Jacobs, Harold. *Geometry.* W.H. Freeman and Company, 1974.

Johnson, Donovan A. *Paper Folding Geometry.* J. Weston Walch, 1976.

Juster, Norton. *The Dot and the Line.* Random House, 1963.

Kirk, Jim. *Environmental Geometry.* Activity Resources Co., Inc., 1972.

Krause, Eugene F. *Taxicab Geometry.* Addison-Wesley, 1975.

Laycock, Mary, and Smart, Margaret. *Focus on Geometry,* Activity Resource Co., Inc., 1982

Laycock, Mary. *Straw Polyhedra.* Creative Publications, 1970.

Laycock, Mary, and Johnson, Connie. *The Tapestry of Mathematics.* Activity Resources Company, Inc., 1978.

Laycock, Mary and Watson, Gene. *The Fabric of Mathematics.* Activity Resources Company, Inc., 1975.

National Council of Teachers of Mathematics. *Geometry in the Mathematics Curriculum.* Yearbook, 1973.

National Council of Teachers of Mathematics. *Geometry Problems My Students Have Written.* Edited by Seaton E. Smith, Jr. and Carl A. Backman. NCTM, 1979.

National Council of Teachers of Mathematics. *Polyhedron Models for the Classroom.* NCTM, 1975.

National Council of Teachers of Mathematics. *Readings in Geometry from the Arithmetic Teacher.* NCTM, 1970.

National Council of Teachers of Mathematics. *Topics in Mathematics for Elementary School Teachers,* Booklet 14, Informal Geometry. NCTM, 1969.

National Council of Teachers of Mathematics. *Topics in Mathematics for Elementary School Teachers, Booklet 18, Symmetry, Congruence, and Similarity.* NCTM, 1969.

O'Daffer, P., and Clemens, S. *Geometry: An Investigative Approach.* Addison Wesley, 1976.

__________. *Laboratory Investigations in Geometry.* Addison Wesley Publishing Company, paperback, 1976.

Olson, Alton. *Mathematics Through Paper Folding.* National Council of Teachers of Mathematics, 1977.

Schaaf, Oscar, ed. *Problem Solving in Mathematics,* Lane County Mathematics Project, Dale Seymour Publication, 1983.

Silvey, Linda. *Geo-Ring Polyhedra.* Creative Publications, Inc., 1972.

Silvey, Linda, and Taylor, Loretta. *Paper and Scissors Polygons.* Creative Publications, Inc., 1976.

Smart, Margaret, and Laycock, Mary. *Focus on Geometry.* Activity Resources Company, Inc., 1982.

Sobel, Max A. and Maletsky, Evan M. *Teaching Mathematics: A Sourcebook of Aids, Activities, and Strategies.* Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1975.

Stokes, William T. *Gems of Geometry.* Stokes, 1978.

Walter, Marion T. *Boxes, Squares, and Other Things.* National Council of Teachers of Mathematics, 1970.

Wood, E. Anne. *Applications in Geometry* and Jean Hanson, *There is Geometry Before Junior High.* Workshop packets from Mathematics Plus, a Title IVC project, ESC Region XIII.

1[illegible]

Suggested Materials for Teaching Advanced Mathematics for Business

Advanced Mathematics for Business is not listed on the textbook adoption cycle. Materials to teach the course will need to be purchased with local funds.

One textbook that may be appropriately matched with the essential elements is:

Applied Mathematics for Management, Life, and Social Sciences Barnett and Qiegler, Daellen Publishing Co., 1983.

Resources also may be available from other publishers.

1986

COMPLIANCE STATEMENT

TITLE VI, CIVIL RIGHTS ACT OF 1964; THE MODIFIED COURT ORDER, CIVIL ACTION 5281, FEDERAL DISTRICT COURT, EASTERN DISTRICT OF TEXAS, TYLER DIVISION

Reviews of local education agencies pertaining to compliance with Title VI Civil Rights Act of 1964 and with specific requirements of the Modified Court Order, Civil Action No. 5281, Federal District Court, Eastern District of Texas, Tyler Division are conducted periodically by staff representatives of the Texas Education Agency. These reviews cover at least the following policies and practices:

(1) acceptance policies on student transfers from other school districts;

(2) operation of school bus routes or runs on a non-segregated basis;

(3) nondiscrimination in extracurricular activities and the use of school facilities;

(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

(5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;

(6) nondiscriminatory practices relating to the use of a student's first language; and

(7) evidence of published procedures for hearing complaints and grievances.

In addition to conducting reviews, the Texas Education Agency staff representatives check complaints of discrimination made by a citizen or citizens residing in a school district where it is alleged discriminatory practices have occurred or are occurring.

Where a violation of Title VI of the Civil Rights Act is found, the findings are reported to the Office for Civil Rights, Department of Health, Education and Welfare.

If there is a direct violation of the Court Order in Civil Action No. 5281 that cannot be cleared through negotiation, the sanctions required by the Court Order are applied.

TITLE VII, CIVIL RIGHTS ACT OF 1964; EXECUTIVE ORDERS 11246 AND 11375; TITLE IX, 1973 EDUCATION AMENDMENTS; REHABILITATION ACT OF 1973 AS AMENDED; 1974 AMENDMENTS TO THE WAGE-HOUR LAW EXPANDING THE AGE DISCRIMINATION IN EMPLOYMENT ACT OF 1967; AND VIETNAM ERA VETERANS READJUSTMENT ASSISTANCE ACT OF 1972 AS AMENDED IN 1974.

It is the policy of the Texas Education Agency to comply fully with the nondiscrimination provisions of all federal and state laws and regulations by assuring that no person shall be excluded from consideration for recruitment, selection, appointment, training, promotion, retention, or any other personnel action, or be denied any benefits or participation in any programs or activities which it operates on the grounds of race, religion, color, national origin, sex, handicap, age, or veteran status (except where age, sex, or handicap constitute a bona fide occupational qualification necessary to proper and efficient administration). The Texas Education Agency makes positive efforts to employ and advance in employment all protected groups.

Texas Education Agency
Austin, Texas
CU6 370 06
November 1986

BEST COPY AVAILABLE